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EVIDENCE OF THE NATURE OF ROTATING LIQUID HELIUM*

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Earlier experiments by Hall[1] and also by Walmsley and Lane[2] on the angular momentum content of rotating liquid helium II have involved measurements of torque required to set a vessel of liquid into equilibrium rotation. Integrating the resultant torque-time measurements over the angular acceleration (or deceleration) period yielded thereby a measure of the angular momentum acquired by the helium. Since furthermore an integration over the volume of the vessel was involved, such methods produced composite, over-all data.

The present method[3] permits investigating both the time and position dependence of liquid helium II circulation within a rotating vessel by suspending a movable Rayleigh disk device within such a system. Besides thus providing both instantaneous and local observations, the sensitivity of such a probe to individual fluid component motions permits in fact a still further resolution; the special case of zero response, for example, uniquely specifies each fluid component as stationary.

The liquid helium II sample investigated resides within a cylindrical shaped glass vessel (just under 5 cm inner diameter) rotated axially about the vertical; a small ($\frac{1}{2}$ mm) aperture at the lower (tip) extremity connects the region with the outer helium bath for filling purposes, etc. A "stationary support" is provided inside the container for the disk suspension system which, although not rotating, is equipped (rack and pinion) for position adjustment along the vessel radius. The disk probe (3-mm diameter) is suspended by a quartz fiber (torsion constant ~ 10^{-5} dyne-cm/deg) to present an angle of 45° to the rotational flow direction. Resultant angular deflections from equilibrium orientation (tending to turn the disk across the flow) are observed as displacements of a narrow light beam refected from the (silvered) disk surface and focused upon an optical scale. The present observations are for a vessel rotation rate of 2 rpm.

The results of this investigation include the following principal findings:

(a) A delay interval of several minutes elapses between setting the container suddenly into rotation from rest and initial response of the disk to the liquid helium II system. In contrast, the disk responds immediately to stopping rotation of the vessel. Thus liquid helium II appears to acquire a "rigidity" in rotation which is absent when stationary.

(b) A linear relationship between such delay interval and radial position of the disk indicates that the circulatory motion propagates as a "rotation front" slowly inward from the moving wall of the container with a definite velocity.

(c) Steady-state angular deflections, reached after roughly one-half hour buildup time from initial response, obey a parabolic dependence upon radial distance from the axis.

(d) Finally, steady-state deflections depend markedly upon temperature. Upon cooling through the λ point the apparent rotational flow drops abruptly from the helium I value to zero, indicating no circulation at all. With reduction of temperature further into the helium II region circulation reappears, however, and builds up mono-

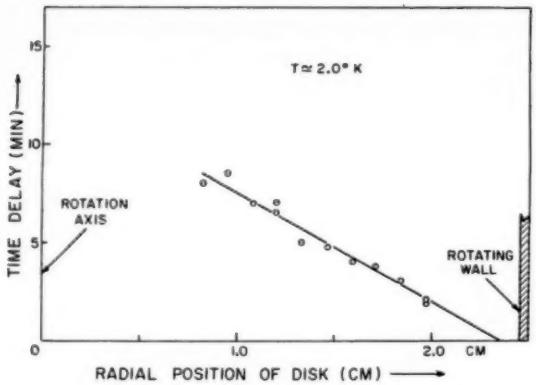

FIG. 1. Time delay versus disk location. The time interval (min) preceding initial response is plotted versus distance (cm) from rotation axis to midpoint of disk. Note location of intercept.

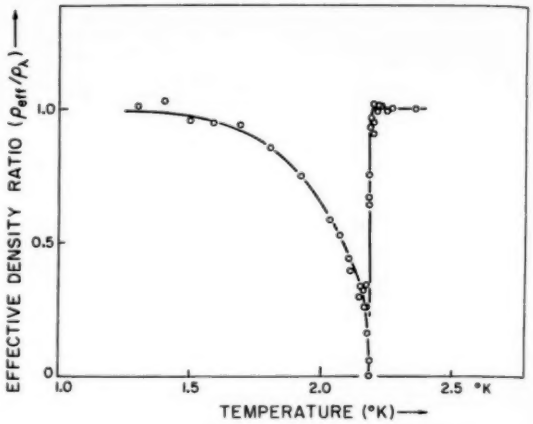

FIG. 2. Temperature dependence of fluid rotation. The effective density ratio (ρ_{eff}/ρ_λ), plotted versus temperature (°K) for a fixed disk location (1.84 cm from axis), represents the fraction of fluid carried into rotation with vessel. Note apparent rejection of fluid rotation upon cooling through λ point.

tonically to the helium I value toward the lower temperature limit ($T - 1$°K).

The "rotation front" propagation phenomenon (b) is illustrated in Fig. 1, where response delay times (min) are plotted versus radial distance (cm) to the disk midpoint from the rotation axis. For a temperature of 2.0°K (and vessel rotation rate of 2 rpm), the observed slope indicates an apparent "rotation front" velocity radially inward of about 2 mm/sec. Furthermore, observation that the intercept (zero time lapse) falls inside the liquid helium region indicates the liquid "circulation front" first disturbs the extreme edge of the disk.

The unexpected temperature dependence (d) is illustrated in Fig. 2, where fluid circulation is plotted versus temperature (°K) for a fixed disk location (1.84 cm from axis of rotation). Observed results are expressed most directly as an effective density ratio (ρ_{eff}/ρ_λ), representing the fraction of fluid evidently carried into rotation with the vessel; the ratio measures the observed torque normalized to the helium I value (at the λ point). Clearly a marked dependence of temperature exists, characterized by a sharp drop evidently to zero circulation immediately below the λ point. A general (approximate) resemblance may be noted between this curve for effective density ratio and the well-known temperature dependence of superfluid concentration (ρ_s/ρ). Such behavior might be interpreted (but possibly oversimplified) to indicate that under these circumstances superfluid rotates whereas normal fluid does not.

Logical explanations for findings (a), (b), and (c) follow directly in principle from an application[4] of the Feynman concept[5] of rotating liquid helium II, based on migrating quantized vortex distributions. Observed temperature dependence of finding (d) appears unaccountable, however, on the basis of either theory or (previous) experiment. According to Feynman's theory[5] and Hall's measurements,[1] for example, indicating rotation of both fluid components with a containing vessel, no temperature effect might be expected. According to Walmsley and Lane's measurements,[2] on the other hand, indicating rotation of normal fluid alone, an increase of circulation with temperature might be anticipated. The actual observed behavior remains clearly oblique to such expectations. The possibility naturally exists at this juncture that some obscure phenomenon related to the method of measurement is responsible.

A paper now in preparation to describe fully the experiment and results will be submitted shortly.

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FIRST-ORDER TERRESTRIAL ETHER DRIFT EXPERIMENT USING THE MÖSSBAUER RADIATION

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Despite the general acceptance of the special theory of relativity, the Fitzgerald-Lorentz contraction theory and its ability to account for previous experiments have persisted. In particular, the extension of the Fitzgerald-Lorentz contraction theory by Ives[1] appears to bring the contraction theory on a par with both the special and general theories of relativity. It has been pointed out[2] that this impasse may be resolved by astronomical observations and satellite experiments designed to detect a variation in the one-way velocity of light with direction in space. The existence of such an observed variation, which may be considered to be synonymous with the existence of an ether, is allowed by the Fitzgerald-Lorentz contraction theory but not by the special theory. This note proposes a terrestrial experiment which has considerable sensitivity when the Mössbauer radiation is used and which permits a first-order test for a variation in the velocity of light with direction in space.

An electromagnetic radiator having a highly stable frequency, a frequency-selective absorber or filter, and a counter or other suitable detector are placed on a turntable, as shown in Fig. 1. Electrical connections to the counter or detector may be made through slip rings. If there is an ether and the velocity of the plane of the turntable through the ether is v, then the time taken for the radiation to travel the distance s between radiator and absorber is

$$\tau = \frac{s}{c - v\cos\theta}, \qquad (1)$$

where c is the velocity of the radiation in the medium between radiator and absorber. When the table is rotated there is a variation in this time if $v \neq 0$. As the time varies there is a change, $-f\,d\tau$, in the phase of the radiation as received at the absorber, where f is the frequency of the radiation. The time rate of change of the phase corresponds to a change in frequency.

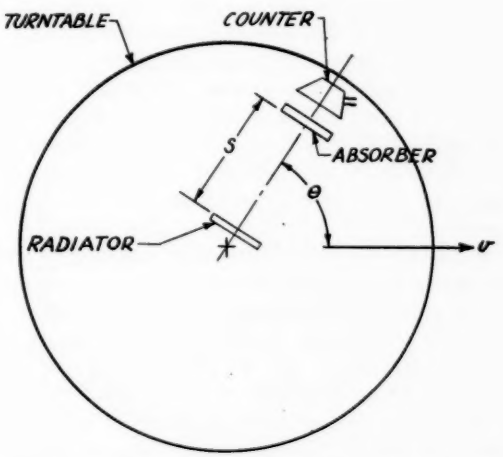

FIG. 1. Arrangement for the detection of a one-way variation in the observed velocity of light.

The relative change in frequency at the absorber, when $v \ll c$, is then, to first order,

$$\frac{\Delta f}{f} = -\frac{d\tau}{dt} = \frac{\omega s v \sin\theta}{c^2} - \frac{v}{c^2}\frac{ds}{dt}\cos\theta - \frac{1}{c}\frac{ds}{dt}, \qquad (2)$$

where ω is $d\theta/dt$, the angular velocity of the turntable. The last term is the Doppler shift. It and the second term may be minimized by using a rigid construction to insure that ds/dt is substantially zero.

If the linewidth of 1 part in 10^{12} achievable with the Mössbauer radiation[3] is used for detecting a minimum change in frequency, s is made $1/10^4$ km and is constant, and ω is 60π rad/sec, corresponding to 1800 rpm, then an ether drift of about 5 km/sec is measurable. With a "least count" sensitivity of 3 parts in 10^{16} estimated for the Mössbauer radiation[4] and the above values for s and ω, an ether drift of about 0.0015 km/sec is measurable.

The axis of the table may be located anywhere

along the measurement axis to permit a rotationally balanced structure. The table may be placed on gimbals to allow adjustment of the angle between the plane of the table and v. To remove an ambiguity in the direction of v, which results when the absorber and radiator are nominally resonant at the same frequency, their resonant frequencies should be separated. It is expedient to make the separation such that maximum sensitivity is obtained, i.e., when the resonant frequency of radiator or absorber is along the linear portion of the resonance curve of the other. The separation may be accomplished, for example, with a Doppler shift, by mounting the radiator or absorber on a loudspeaker movement or similar transducer and providing a sawtooth motion of proper amplitude and phase and of the same period as that of the table rotation. A difference in temperature between radiator and absorber[5] may also be used to separate the resonant frequencies.

Since this experiment is a first-order test, second-order effects are neglected. These include the transverse Doppler shift, Lorentzian variations in mass, length, and time, the bending and frequency shift of light in a gravitational field, and any anisotropy of space of gravitational origin. Only if a negative first-order effect is obtained will it be reasonable to use the proposed experimental arrangement to search for second-order effects.

Parenthetically, the resemblance between the proposed experiment and the terrestrial measurement[6] of the red shift in an accelerated system is superficial. In the latter, the counter is fixed near the periphery of the table and measures the difference in frequency between radiator and absorber for only one orientation of the table in space. If there is an ether, a frequency shift would result from the daily rotation of this particular orientation, but it would be very small, and probably immeasurable, due to the low angular velocity of the earth. If the radiation between radiator and absorber were not perfectly parallel, it would, in the presence of an ether, increase the error in the red shift measurement, but the effect would be small and difficult to interpret. To detect sensibly a variation in the frequency difference between radiator and absorber resulting from an anisotropy in the observed velocity of light it is expedient to monitor the difference frequency as the orientation of the radiator-absorber axis changes. To accomplish this in lieu of rotating the counter it is feasible to add stationary counters around the periphery of the table and to compare their outputs simultaneously.

This experiment, because it is a one-way test, is uniquely capable of differentiating between the special and contraction theories. In the contraction theory, the observed velocity of light, to the first order, is the denominator of Eq. (1), i.e., $c - v\cos\theta$. The special theory is correct and there is no Lorentz (optical) ether only if $v = 0$. If $v \neq 0$, the observed velocity of light varies with angle θ in contradiction with the second postulate of the special theory, only the contraction theory can be correct, and a Lorentz ether (which should not be confused with a modern ether attributable to a gravitational anisotropy) then physically exists. All previous experiments have been measurements of a two-way light signal or the equivalent. The measured two-way velocity in the contraction theory, to first order, is the average of $c \pm v|\cos\theta|$, or just c, as predicted by the special theory. The cancellation of the first-order terms has always produced an identical result (to the second order) for the contraction and special theories. Thus, the two-way experiments, as has been intermittently noted in the past, are not capable of measuring v or of detecting a Lorentz ether. A one-way test, which has never been knowingly performed on earth heretofore, may thus be termed a crucial experiment for it provides a final resolution of the issue between the relativity and contraction theories.

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COLLECTIVE OSCILLATIONS AND GIANT DENSITY FLUCTUATIONS*

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It is the purpose of this note to interpret an instability in compressional modes found by Glassgold, Heckrotte, and Watson (GHW).[1] We show that their instability is related to the Hartree-Fock (HF) treatment of many-body systems by Overhauser,[2] who claims that HF theory always predicts a ground state in a Fermi gas with a stationary density fluctuation of a given wavelength present. Kohn and Nettel (KN)[3] give a demonstration that for three dimensions this cannot in general be the case for weak interactions. The present work is in support of KN. However, in addition to their negative result, we find that in a linearized theory, a sufficiently strong attractive interaction gives rise to stable HF solutions with a built-in density ripple, as well as a GHW blow-up. For either repulsive or weak attractive interactions there is neither instability in GHW nor a self-consistent density ripple. This is in complete analogy to the elegant analysis of Wolff[4] on the spin density case. In neither of these cases is one involved in a BCS type transition which takes place for infinitesimal attractive interactions.[5] Further the present method seems inapplicable to study the well-known lattice type transition of the low-density electron gas. A possible application, discussed at the end of this note, is the theory of freezing of He^3.

Our starting point is Overhauser's SCF analysis.[6] Since in this work we discuss density fluctuations we shall, as do KN, confine ourselves to the Hartree field alone. This is permissible if spin is unimportant and brings out the qualitative aspects of the situation. The Schrödinger equation for a state $\vec{k}$ with energy $E_{\vec{k}}$ and wave function $\phi_{\vec{k}}$ (normalized in a box Ω) in Hartree theory is

$$[-(\hbar^2/2m)\nabla^2 + V(\vec{r})]\phi_{\vec{k}} = E_{\vec{k}}\phi_{\vec{k}}, \tag{1}$$

$$V(\vec{r}) = \sum_{\vec{k}} \int \phi_{\vec{k}}^*(\vec{r}')v(\vec{r}-\vec{r}')\phi_{\vec{k}}(\vec{r}')d^3r'. \tag{2}$$

Here $v(\vec{r})$ is a two-body potential assumed to have a range. With Overhauser we search for density fluctuations of wave number $\vec{q}$; i.e., $V(\vec{r})$ has only a single Fourier component. In perturbation theory, we have

$$\phi_{\vec{k}} = \Omega^{-1/2}\left\{e^{i\vec{k}\cdot\vec{r}} + [g^+/\omega_{\vec{q}}(\vec{k})]e^{i(\vec{k}+\vec{q})\cdot\vec{r}} + [g^-/\omega_{-\vec{q}}(\vec{k})]e^{i(\vec{k}-\vec{q})\cdot\vec{r}}\right\}, \tag{3}$$

$$\omega_{\vec{q}}(\vec{k}) = [(\vec{k}+\vec{q})^2 - \vec{k}^2]/2m, \tag{4}$$

$$g^{\pm} = \int e^{-i(\vec{k}\pm\vec{q})\cdot\vec{r}} V(\vec{r}) e^{i\vec{k}\cdot\vec{r}} d^3r = \int e^{\pm i\vec{q}\cdot\vec{r}} V(\vec{r}) d^3r. \tag{5}$$

Substituting the solution (3) with the definitions (4) and (5) into Eq. (2) and carrying linear terms in $g^+(\vec{k})$ only (which is consistent with our general linear development), we have

$$g^+ = [v(\vec{q})/\Omega]\sum_{\vec{k}'}[g^-/\omega_{-\vec{q}}(\vec{k}') + g^+/\omega_{\vec{q}}(\vec{k}')]. \tag{6}$$

Multiplying Eq. (6) by $1/\omega_{\vec{q}}(\vec{k})$, combining with the equation for $g^-(\vec{k})$, and summing on $\vec{k}$ gives the necessary condition that a nontrivial solution exists:

$$1 = [v(\vec{q})/\Omega]\sum_{\vec{k}}{}'[1/\omega_{\vec{q}}(\vec{k}) + 1/\omega_{-\vec{q}}(\vec{k})]. \tag{7}$$

The sum on $\vec{k}$ is for $|\vec{k}|<k_F$; $|\vec{k}+\vec{q}|>k_F$; k_F = Fermi momentum; $v(\vec{q}) = \int v(\vec{r})e^{i\vec{q}\cdot\vec{r}}d^3r$. With this remark Eq. (7) is transformed to

$$1 = [v(\vec{q})/\Omega]\sum_{\vec{k}}\{[n(\vec{k}+\vec{q}) - n(\vec{k})]/\omega_{\vec{q}}(\vec{k})\}, \tag{8}$$

which is the same as KN Eq. (12).

Before we discuss Eq. (8), we first summarize the theory of collective oscillations in random phase approximation (RPA). A convenient approach is the dielectric constant method.[7] In response to a driving force of time dependence $e^{i\omega t}e^{-\epsilon|t|}$, the "dielectric constant," defined as the ratio of induced density fluctuation to driving force, in RPA is

$$\epsilon_{\vec{q}}(\omega) = 1 + 4\pi\alpha_{\vec{q}}(\omega) = 1 - [v(\vec{q})/\Omega]\sum_{\vec{k}}\frac{n(\vec{k}+\vec{q}) - n(\vec{k})}{\omega_{\vec{q}}(\vec{k}) - \omega + i\epsilon}. \tag{9}$$

The root of $\epsilon_{\vec{q}}(\omega)$ gives the frequency $\omega(\vec{q})$ for

which waves propagate freely in the system. In RPA, these, generally, will not be damped.

For $q \ll k_F$, the function $4\pi\alpha_{\vec{q}}(\omega)$ takes on the simple well-known form

$$4\pi\alpha_{\vec{q}}(\omega) = -v(q)g(\epsilon_F)\left[1+(\xi/2)\ln\left|\frac{1-\xi}{1+\xi}\right| - \frac{\pi i}{2}\xi[1-\theta(\xi)]\right], \tag{10}$$

where $v_F = k_F/m$. The term $[(\pi i/2)\xi]$ arises for $\xi < 1$ in the limit $\epsilon \to 0$, hence the step function $[1-\theta(\xi)]$; $g(\epsilon_F) = [\text{density of states at the Fermi surface}]/\Omega$; $\xi = (\omega/v_F q)$. In Fig. 1, the function $-\mathrm{Re}[4\pi\alpha_{\vec{q}}(\xi)/v(q)g(\epsilon_F)]$ is plotted for real ξ. In the figure three situations are indicated. For repulsive forces, there is always a solution with $\mathrm{Re}\xi > 1$; $\mathrm{Im}\xi = 0$. These are plasmons for the case of Coulomb forces and "zero sound" for the case of forces with range. For attractive interactions with $g(\epsilon_F)|v(q)| < 1$, no root exists and no free oscillation occurs. For attractive forces with $g(\epsilon_F)|v(q)| > 1$, if a solution exists it would have $\mathrm{Re}\xi < 1$ in which case we have to include the imaginary term in Eq. (10). In this case the solution in both the above examples is easily seen to be $\mathrm{Re}\xi = 0$; $\mathrm{Im}\xi = [1 + 1/gv]$. In particular the sign of $\mathrm{Im}\xi$ changes as gv passes through -1 with the stable side being for weak interactions ($g|v| < 1$). It is thus found that a critical attractive force exists ($|v(q)| = [g(\epsilon_F)]^{-1}$) in order for instability to occur. This is identical to the case of exchange interactions where a critical ferromagnetic exchange force was necessary for a ferromagnetic instability.[4]

Comparison with Eq. (8) then shows that it cannot be a consistent equation unless $g(\epsilon_F)v(q) = -1$. For $v(q)$ repulsive or $g(\epsilon_F)|v(q)| < 1$, the only solution of Eq. (6) is $g^{\pm}(k) = 0$. For $g(\epsilon_F)v(q) < -1$, a Hartree solution with a density ripple occurs.

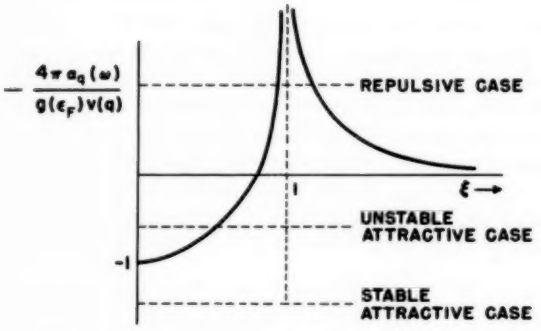

FIG. 1. Plot of polarizability in dimensionless units vs frequency in dimensionless units.

It is manifested by a blow-up in zero sound about the spherical Fermi sea and according to Overhauser[6] has a lower energy than the latter.

A possible application of the above is to note that when He^3 is put under pressure, there is a certain critical pressure (~30 atm) for which it freezes. Now He^3 is rather an open liquid with [interparticle distance] >> [position in minimum of $v(r)$]. A careful calculation (using "t" matrices for v matrices) should then show how $v(q)$ grows with pressure. Presumably this will occur fastest for $q \approx [\text{interparticle distance}]^{-1}$. At the point of instability, the gas-like quasi-particle picture will break down and freezing will set in.

One last comment on the mathematics of the instability is in order. There are cases when collective modes have complex frequencies with imaginary parts which damp the oscillation even in RPA. A case in point is phonons in metals.[8] Here for small q the phonon is given by the roots of

$$\epsilon_{\vec{q}}(\omega) = 1 - \omega_M{}^2/\omega^2 - 4\pi\alpha_{\vec{q}}(\omega), \tag{11}$$

where $\omega_M{}^2 = 4\pi ne^2/M$; M = ion mass; n = ion density; $\alpha_{\vec{q}}(\omega)$ = electron polarizability. One finds a root at $\mathrm{Re}\omega = cq$, where $c = (m/3M)^{1/2}v_F \ll v_F$. Hence this root has $\xi \ll 1$. Nevertheless it is damped rather than divergent. This is because the sign of the imaginary part is determined by the derivative of $\epsilon_{\vec{q}}(\omega)$ at the root[9] which in the present case is determined by the $\omega_M{}^2/\omega^2$ term in (11). In the previous case it was determined by $\alpha_{\vec{q}}(\omega)$.

Finally, it should be pointed out that Thouless[10] and Pomeranchuk[11] have given general criteria for the stability of HF solutions which are no doubt related to the topics discussed here.

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ABSORPTION SPECTRA AND ZEEMAN EFFECT OF COPPER AND ZINC IMPURITIES IN GERMANIUM*

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The absorption spectra of group V and group III impurities in germanium have been reported previously.[1,2] For these impurities of small ionization energy, theoretical treatment based on the effective-mass formalism should be a good approximation, and the experimental results, especially in the case of donors, bear this out. Experimental investigation has been now extended to two acceptor impurities, Cu and Zn, of higher ionization energies. The ionization absorption of Cu and Zn in germanium has been seen before[3,4] but no observation of the excitation lines has been reported. Each of these impurities can bind more than one hole. Studies have been made on the neutral and the singly charged states. It may be expected that the effective-mass formalism would not apply for the ground state but may still be valid for the excited states. The absorption spectra provide the means to check this assumption and to reveal effects due to differences in the ground states.

Measurements were made at liquid helium temperature. For the observation of the Cu^- and Zn^- absorptions, samples suitably compensated by Sb impurity were used. The solid curve in Fig. 1 shows the absorption spectrum of Cu. The observed lines are labeled according to the designation used for the spectra of group III impurities.[2] As seen in Table I, the energy spacings between the *D* line and the various other lines are very close to the corresponding spacings in the group III spectra. This applies also to the spectrum observed for Zn, the data for which are included in Table I. In the case of Zn, addi-

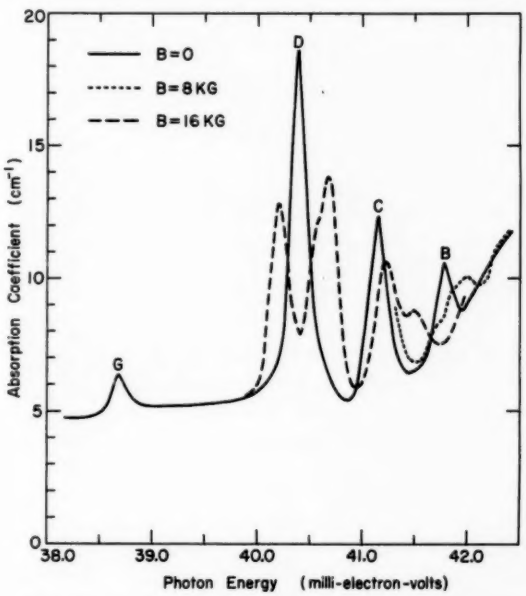

FIG. 1. Absorption spectrum and transverse Zeeman effect of Cu impurity in germanium. Copper concentration is 1.2×10^{16} cm^{-3}. Magnetic field is parallel to [111] in the (110) plane.

Table I. Ionization values and energy spacings between the excitation lines for various impurities in germanium, in units of 10^{-3} ev.

	Group III (average values)	Cu	Zn	Zn^-
B-D	1.388	1.398 ± 0.020	1.382 ± 0.051	···
C-D	0.722	0.745 ± 0.016	0.729 ± 0.033	3.33 ± 0.06
D-G	1.73	1.72 ± 0.03	1.84 ± 0.03	···
Ionization energies	Optical	42.8	32.6	85.8
	Thermal	40	30	90

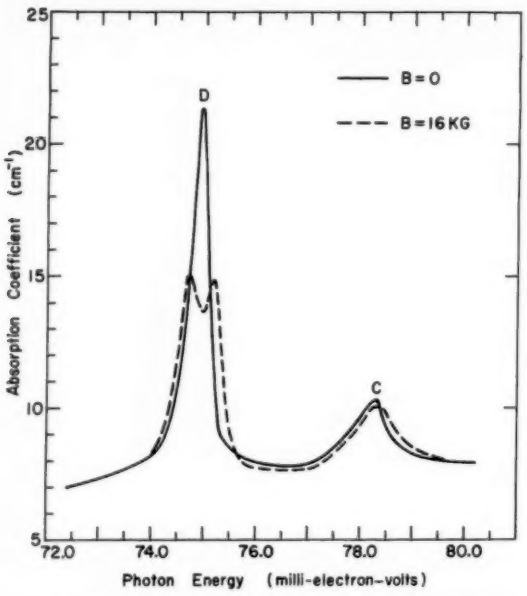

FIG. 2. Absorption spectrum and transverse Zeeman effect of Zn^- impurity in germanium. The sample is compensated with estimated 3.8×10^{15} cm^{-3} Zn and 2.2×10^{15} cm^{-3} Sb. Magnetic field is parallel to [111] in the (110) plane.

tional structure was observed on the long-wavelength side of the G line, the explanation for which is not yet clear. The solid curve in Fig. 2 shows the spectrum for Zn^-. Only two lines were observed which were identified with the two prominent lines C and D of the other spectra. The spacing between these two lines, as given in Table I, is about 4.6 times the corresponding spacing of the other acceptors studied. Strictly on the basis of effective-mass formalism, Zn^-, having a doubly charged center, is expected to have spacings between the excited states which are 4 times larger than those for a singly charged center. However, the excited states concerned in the cases of the C and D lines of Zn^- have binding energies already comparable to the binding energies of the ground states of group III impurities. A somewhat larger spacing between the C and D lines of Zn^- is not unexpected. In the case of Cu^-, we have not yet seen the excitation lines. However, indications of the ionization absorption have been observed.

In the case of group III impurities, we have estimated the ionization energy by adding to the energy of the A line (ground state to $2p^4$ transition) the theoretically calculated binding energy of the $2p^4$ state, 0.0006 ev.[5] This procedure minimizes possible error in the theoretical estimate as the $2p^4$ state is the highest excited state calculated. The A line has not yet been observed for Cu and Zn. The ionization energies for these impurities may be estimated by adding to the energy of the D line the binding energy of the $2p^2$ state which was obtained from the spectra of the group III impurities. Since the C-D spacing is 4.6 times larger for Zn^-, we may estimate the ionization energy by adding to the D line a proportionately larger binding energy. The value obtained is close to the reported thermal ionization energy, but must be regarded as only an approximate estimate.

The effect of the magnetic field on the excitation lines has been investigated. The observations confirm the assignment of the lines. The dashed curves in Figs. 1 and 2 show the results obtained at a field of 16 kgauss. The D line splits into two components in all cases, Cu, Zn, and Zn^-. The splitting is proportional to the magnetic field and is comparable to that observed for the D line of boron impurity. For Cu, Zn, and B, the high-energy component appears to be broader, and there is indication that this component may further split. Studies with higher fields and polarized light are in preparation. Such measurements reported recently for aluminum in silicon[6] show rather complex Zeeman effects. The situation in silicon is complicated by the fact that the separation of the third valence band by the spin-orbit interaction is very small.

The C line in the Cu and Zn spectra broadens and shifts to higher energy. It resolves into two peaks at higher magnetic fields. The separation of each peak from the zero-field position of the line appears to increase quadratically with the field. The effect appears to be identical with that of the C line of boron. The Zeeman effect of the C and D lines reinforces the conclusion that the excited states of Cu and Zn are similar to those of the group III impurities, and indicates that the observed effect comes mainly from the excited states. In the case of Zn^-, the shift of the C line is much smaller as can be seen by comparing Figs. 1 and 2. This is as expected. With a doubly charged center, the dimensions of the wave functions of Zn^- should be about one-half of that for singly charged centers. The quadratic Zeeman effect being a measure of the area of the wave functions, it might be expected to be about four times smaller for Zn^-. This is consistent with

the observation.

It is interesting to note that the absorption lines of impurities are very much sharper in germanium than in silicon. The full width at half absorption is about 0.19×10^{-3} ev for the observed lines of Cu and the group III impurities. The actual width is probably even smaller, since the observed width is comparable to the resolution used. This is to be compared with the width of 1×10^{-3} ev reported for boron lines in silicon.[7] The broadening of the absorption lines has been treated[7,8] theoretically on the basis of the scattering of the bound hole by the acoustical vibration of the lattice. According to these treatments which used the hydrogenic approximation for the impurity, the width is determined mainly by the broadening of the ground state, and for germanium and silicon the ratio of the widths is expected to be of the order of the reciprocal of the Bohr radius squared. On this basis, we would expect the lines of the group III impurities to be 6 to 7 times narrower in germanium than in silicon. The observation shows that the ratio of the linewidths in these two materials is at least as large as this estimate. The lines of Zn^- are considerably broader, about 0.43×10^{-3} ev in width for the D line. The width is considerably larger than the instrumental resolution. This result further substantiates the expectation that the broadening of the ground state increases with decreasing orbit dimension. In this connection, the failure to observe the excitation lines of Cu^- might be caused by large broadening. The orbit of the hole in the ground state may be quite small for Cu as indicated by the large ionization energy of 0.33 ev.

One additional point of interest is the difference in the relative strengths of the various lines for different impurities. This is particularly pronounced in comparing the spectrum of Zn with that of the group III impurities. The C line is much weaker than the D line in the case of Zn^-. Noticeable variations can be seen even among Cu, Zn, and the group III impurities, which should reflect mainly the difference in the ground states. Accurate data have yet to be obtained.

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HYPERFINE STRUCTURE OF THE F CENTER IN LiF*

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(Received August 1, 1960)

The F center in LiF is one of the color centers in alkali halides which has been most throughly studied experimentally by magnetic resonance[1-8] and theoretically.[9] To date there is serious disagreement between experiment and theory. Lord[2] originally reported resolved structure of the electron spin resonance, which he interpeted as arising from the hyperfine coupling of the electron to the six Li^7 atoms nearest to the vacancy. Although he failed to see all of the 19 lines predicted by this model, he later supported his contention[3] with the direct measurement of the hyperfine constants by the electron-nuclear double resonance technique (ENDOR).[10] Stimulated by this work, Gourary and Adrian[9] calculated the hyperfine coupling with the nearby nuclei. While the agreement with the nearest lithium atoms was satisfactory, the prediction for the nearest fluorine atoms was too large by a factor of 30. The picture was further complicated when Kim, Kaplan, and Bray[4] reported that they could resolve at least 10 more lines than the 19 predicted. Hyde[8]

has even suggested that the resolved structure is not due to the F center.

We have investigated the paramagnetic resonance of x-rayed LiF crystals containing F and M centers. This Letter reports the observation of 35 lines in electron spin resonance (ESR), together with new electron-nuclear double resonance data which enable us to predict the observed electron spin resonance in great detail and to resolve the discrepancy between theory and experiment.

The samples were obtained from Harshaw, and either used without treatment, or annealed in an inert atmosphere, or left exposed to the atmosphere for over a year. They were then exposed to 50-kv x rays at room temperature for varying lengths of time ranging from 1/2 hour to 15 hours. Optical density (OD) measurements show that the F-center concentration is varied from $10^{17}/cm^3$ to greater than $10^{18}/cm^3$ while the ratio OD_F/OD_M is varied from 600 to ~3.5. The electron spin resonance is observed both at room temperature and at 1.3°K. The crystal is located midway on the narrow side wall of a TE 101 mode microwave cavity which is slit vertically on the center line of the broad face, the two halves electrically insulated to allow rf penetration from the rf coil wound on the exterior of the cavity for ENDOR experiments at 1.3°K. The X-band paramagnetic resonance spectrometer consists of a transmitter klystron employing a modified Pound i.f. frequency stabilization scheme and a high-sensitivity superheterodyne receiver.

With H_0 parallel to the [100] direction, the electron spin resonances of all specimens exhibit 35 lines with a spacing of ~14 gauss, uniformly decreasing in amplitude from the center of the line and symmetric about $g = 2.0006 \pm 0.0006$. A typical slow-passage derivative record is shown in Fig. 1. To determine whether or not two resonances are present, one broad and structureless, the other possessing structure, we have checked the ratio of the resolved structure amplitude to the broad resonance amplitude of each sample. It is constant to within a factor of 2, thus essentially independent of the F/M concentration ratio for the range of concentrations we have used. The resolution is improved tenfold when H_0 is parallel to [111] and decreased twofold when H_0 is parallel to [110]. The over-all shape of the integrated experimental curve is nearly Gaussian with a width of 85 ± 10 gauss from peak to center of the derivative.

An attempt to explain the excessive number of lines by considering either forbidden transitions or an isotope effect is not successful when one uses Lord's values of the hyperfine couplings.

Electron-nuclear double resonance signals can be grouped, each group arising from a shell of nuclear sites which are crystallographically equivalent with respect to the F-center negative ion vacancy and whose distance from the vacancy is $\sqrt{N}$ times the lattice parameter, where N is an integer. For a given shell N, there are two sets of ENDOR lines arising from cases where the field of the electron at the nucleus either aids

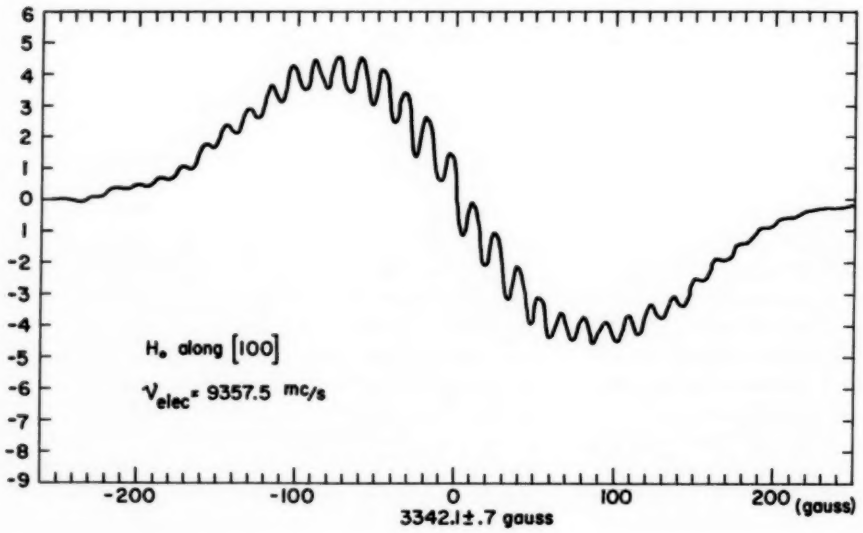

FIG. 1. ESR absorption derivative curve showing 35 resolved lines. H_{mod} = 2 gauss at 800 cps. T = 1.3°K. $H_{microwave} \sim 3 \times 10^{-6}$ gauss.

or opposes the external field. (We call the two cases respectively "up" and "down.") To check that lines were properly identified as "up" or "down," we have studied the variation of their frequency with external field. We verified that the Shell 1 lines were correctly identified. A search for high-frequency ENDOR lines not previously reported, however, reveals a complete set of lines having the symmetry of Shell 2, the nearest fluorines. The nuclei giving rise to these lines are identified as fluorines from the dependence of the ENDOR frequency on H_0 and by the separation of the "ups" and "downs" by twice the Larmor frequency in the static field (see Fig. 2). From the magnitude of the hyperfine splitting we conclude these are in fact Shell 2 and that Lord's Shell 2 lines are Shell 8. The ENDOR pattern is usually computed using first-order perturbation theory. If the hyperfine coupling is treated to second order, subsidiary splittings arise.[11] Such a set is illustrated by the 4 lines near 31 Mc/sec which arise from the Shell 2 fluorines. The calculation predicts a separation of 0.5 Mc/sec with the intensity ratios 1:4:4:1 as observed. In addition, the ENDOR lines were observed to obey the correct angular dependence of Shell 2 when the crystal is rotated about a [110] axis as well as a [100] axis perpendicular to H_0.

ENDOR lines are identified through Shell 8 (there is no Shell 7 by this scheme of numbering shells); the hyperfine constants a and b are given in Table I together with the theoretically predicted values of Gourary and Adrian. The Shell 1 and Shell 2 values are in rough agreement with the predicted values.

Table I. Hyperfine constants in megacycles/sec for the first seven shells of nuclei surrounding the F center in LiF.

Shell	Nucleus	a	b
1	Li^7	39.06 ± 0.02 (theo. = 50)	3.20 ± 0.01 (theo. = 2.8)
2	F^{19}	104.94 ± 0.02 (theo. = 61)	14.96 ± 0.02
3	Li^7	0.50 ± 0.01	0.68 ± 0.02
4	F^{19}	0.48 ± 0.01	1.12 ± 0.01
5	Li^7	0.27 ± 0.01	0.28 ± 0.01
6	F^{19}	0.88 ± 0.01	0.69 ± 0.01
8	F^{19}	1.34 ± 0.01	0.56 ± 0.01

Using the measured Shell 1 and Shell 2 hyperfine values and assuming a rectangular broadening function to take account of the other shells, we have calculated the detailed ESR line shape for H_0 parallel to [111]. The result is shown in Fig. 3 together with an integrated experimental record and a Gaussian. The calculation predicts an average resolved structure separation 6% greater than the Shell 1 a value. This effect is clearly seen in the integrated experimental ESR curve. Both the experimental and calculated curve are nearly Gaussian. The root second moment, calculated from the measured hyperfine constants, is 75.7 gauss; the measured root second moment (from derivative peaks assuming

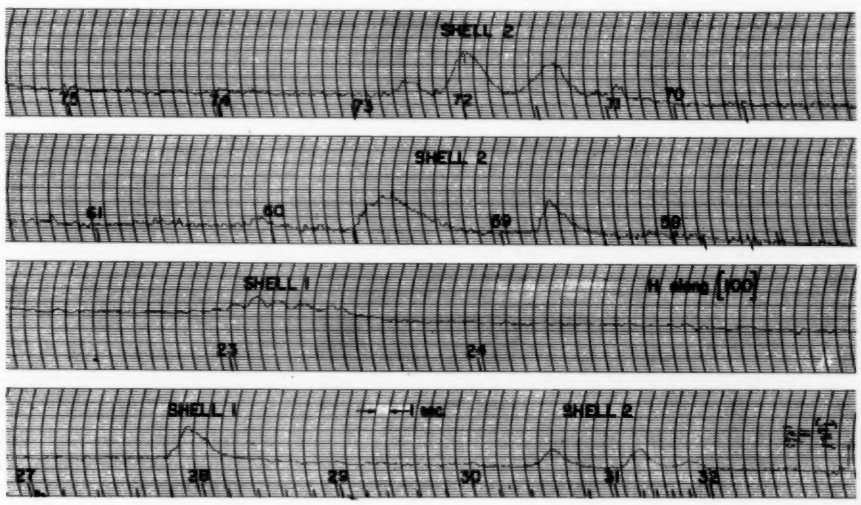

FIG. 2. ENDOR lines showing portions of Shells 1 and 2.

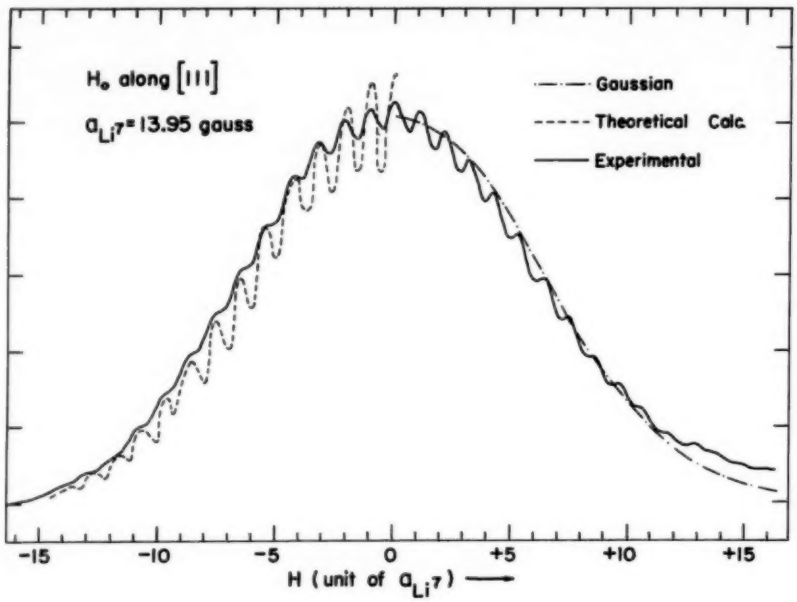

FIG. 3. Comparison of the integral of an experimental ESR curve for H_0 parallel to [111] with the calculated ESR line shape and with a Gaussian curve.

a Gaussian) is 85±10 gauss. We conclude that substantially the entire resonance arises from F centers, that the observed ENDOR lines account completely for the experimental line shape and are in substantial agreement with the calculations of Gourary and Adrian.

Our new increased second moment for the F center is larger than that attributed by Lord both to the F center alone and to the combination of the F and M centers.[12] If we are correct, Lord's detection of the existence of the M-center resonance by second moment methods is inconclusive.

We would like to thank Professor Robert Maurer for making available his Cary spectrophotometer for determination of the optical spectra, and Dr. J. Kingsley for his helpful advice in its use. A substantial amount of the numerical analysis of the double-resonance data was performed by Mr. Nils Fernelius. The numerical computation of the theoretical line shape was performed by Mr. James Bushnell. We wish to thank Dr. George S. Newell who built the microwave spectrometer in conjunction with two of the authors (WCH and HB).

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‖A. P. Sloan Fellow.

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DIFFUSION OF LITHIUM THROUGH TUNGSTEN

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The diffusion coefficients of Li^7 and Li^6 in tungsten have been measured. The lithium is deposited on one surface of a tungsten ribbon and the atoms which diffuse through the tungsten to the opposite face are ionized and identified in a mass spectrometer.

In a typical experiment a circular lithium deposit about 100 atoms thick and 0.3-mm diameter was deposited from a collimated positive-ion beam from a thermal ion source containing the lithium isotopes in the natural proportions. The tungsten ribbon was 1.0 mm wide and 0.030 mm thick. After the deposition the ribbon was electrically heated to a specific temperature in the range 1000°K to 1400°K. The high temperature caused diffusion of the atoms to the other face of the tungsten ribbon which served as the positive-ion source surface for a 60°, 25-cm radius, direction-focussing mass spectrometer. The efficiency of ionization of the atoms reaching the second surface of the ribbon was high, as the work function of the surface was greater than the ionization potential of the diffusing atoms. When the ion current was observed as a function of time, a characteristic curve was obtained as shown in Fig. 1(a).

A solution to the diffusion equation $\partial C/\partial t = D\nabla^2 C$ can be found for this particular arrangement if the diffusion through the filament of thickness L is characterized by a single diffusion coefficient D independent of concentration. It is assumed that the initial deposit is circular of radius b, and of thickness much less than L, and that every atom reaching the second face of the ribbon evaporates from it. The current, I, integrated over the second face at any time t after the diffusion process begins is given by

$$I = A L D^{-1} b^2 t^{-3/2} \exp(-L^2/4Dt),$$

where A is a measure of the initial concentration. From this equation it can be seen that the time to reach the maximum current is $L^2/6D$. D can therefore be found from this time. The value of D can also be found from the slope of the curve obtained when $\log(It^{3/2})$ is plotted against $1/t$. Figure 1(b) shows such a plot for the experimental observations of Fig. 1(a). It can be seen that in the region around the peak current the experimental points lie close to a straight line. The values for D obtained by these two methods agree closely.

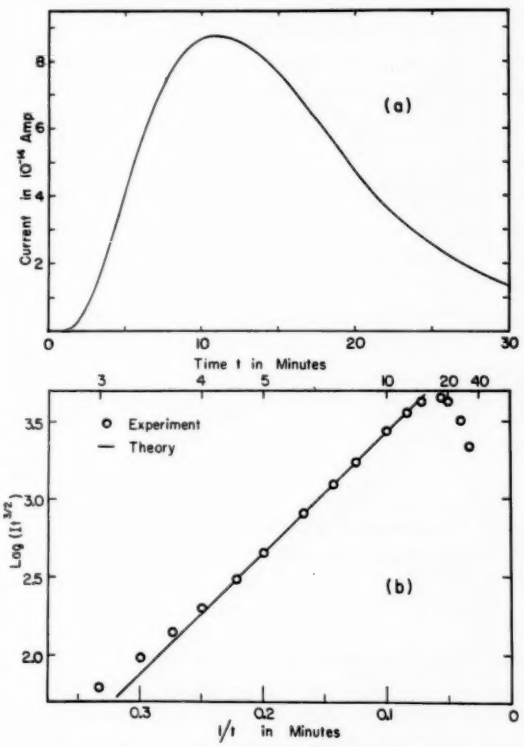

FIG. 1. (a) The current of Li^7 diffusing through a polycrystalline tungsten sample at 1160°K as a function of time. From the time to the peak, the diffusion coefficient, D, is obtained as 2.4×10^{-9} cm^2 sec^{-1}. (b) The same results are now plotted in the form $\log(It^{3/2})$ vs $1/t$ for comparison with the theoretical straight line. The slope gives a value of 2.2×10^{-9} cm^2 sec^{-1} for the diffusion coefficient.

Measurements of the diffusion coefficient as a function of temperature gives the usual temperature dependence $D = D_0 \exp(-\epsilon/kT)$, as seen in Fig. 2, where ϵ, the activation energy, is 2.42 ± 0.05 ev and D_0 for Li^7 is 82 ± 40 cm^2 sec^{-1}.

The values of the diffusion coefficient and of the activation energy indicate that lattice diffusion through the ribbon is being measured rather

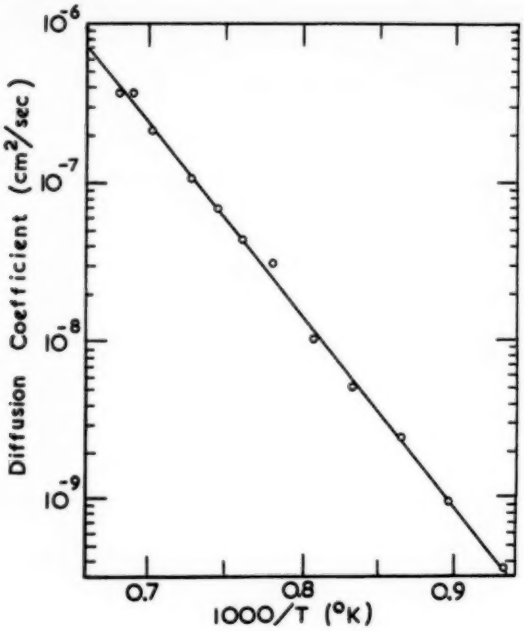

FIG. 2. The diffusion coefficient of Li^7 in polycrystalline tungsten as a function of temperature. The slope gives a value of 2.42 ev for the activation energy.

than grain boundary or surface diffusion. This supposition was confirmed by first varying the ribbon width and then by using a single-crystal ribbon.[1] Results for the diffusion coefficient in both cases were of the same order of magnitude as the original values and within the error in D due to uncertainties in the temperature and thickness of the ribbon. This indicates that lattice diffusion is playing the dominant role.

The discrepancy between the observed and predicted currents at long times is to be expected as the evaporation from the first face is not taken into account in the analysis. The high current at short times, which is observed both with single crystals and with polycrystalline ribbon, could be due either to a surface diffusion contribution or to the diffusion coefficient being dependent on concentration.

This method of measuring diffusion coefficients allows a very simple comparison of the diffusion rates of Li^6 and Li^7. By switching back and forth between the two isotopes during a run, both curves can be obtained by interpolation. An average value of the ratio of the diffusion coefficients obtained was $D_6/D_7 = 1.07 \pm 0.02$ which is in good agreement with the ratio of the inverse square roots of their masses, $(M_7/M_6)^{1/2} = 1.080$. Schoen[2] has shown that the ratio of the diffusion coefficients D_α and D_β of two isotopes of masses M_α and M_β gives an indication of the type of lattice diffusion taking place. The value of $D_\alpha/D_\beta = (M_\beta/M_\alpha)^{1/2}$ is consistent with the interstitial diffusion which is to be expected from the sizes of lithium and tungsten atoms and the tungsten lattice spacing.

We are indebted to Dr. P. A. Puhach of this Department for the theoretical analysis, and to the National Research Council of Canada for financial assistance.

[1]Kindly supplied by C. D. Hartman of the Bell Telephone Laboratories.

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ELECTRIC-MONOPOLE-ENHANCED $2^{+\prime} - 2^+$ TRANSITIONS IN Th^{232} AND U^{238}†

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In the course of a survey for internal conversion electrons from the Coulomb-excited isotopes $Th^{230,232}$ and $U^{234,236,238}$, a number of transitions were observed which are interpreted as electric monopole ($E0$) enhanced transitions between $2^{+\prime}$ beta-vibrational states and 2^+ first excited rotational states. The occurrence of $E0$ enhancement in these cases is in itself important for the identification of the parent Coulomb-excited states as beta-vibrational, since the K-selection rule forbids monopole-enhanced transitions from γ-vibrational states.[1] However, to use these results to test current theories of monopole enhancement[2,3] requires that the $E0$ transition probabilities and the ratio of the $E0$ to $E2$ transition probabilities for these states be determined. To do this it is

necessary to know not only the partial reduced transition probabilities $\epsilon(e^-)B(E2)_d$ given by the present measurements, but also the partial reduced transition probabilities $\epsilon(\gamma)B(E2)_d$. Since the latter results are available at present only for the states in Th^{232} and U^{238}, the following discussion is confined to these cases.

The internal conversion spectra obtained from Th^{232} and U^{238} bombarded with 5-Mev protons are shown in Fig. 1, and the results derived from the measurements are given in Table I. For U^{238} the 991-kev transition is assigned as the $E0$-enhanced $2^{+\prime}-2^+$ cascade from the 1036-kev β-vibrational state. The $E2$ crossover transition to the ground state is not seen. The partial $B(E2)_d$ value derived from these measurements added to that obtained from the γ-ray measurements[4] leads to the tabulated total $B(E2)_d = 2.6B(E2)_{sp}$. Two transitions are seen in the Th^{232} conversion spectrum, a strong monopole-enhanced 725-kev transition and a weak 738-kev transition. Both are assigned as $2^{+\prime}-2^+$, originating in a 775-kev β-vibrational state and a 788-kev γ-vibrational state, respectively.[5] The magnitude of the barely detectable electron yield from the 788-kev transition was estimated also, making it possible to determine $B(E2; 788-50)/B(E2; 788-0)$. With this ratio and the assumption of $E2$ character for the 738-kev transition, the value of $B(E2; 788-0) \simeq 3.8 \times B(E2)_{sp}$ was derived without recourse to any γ-ray data.[6] To obtain the total $B(E2)$ value for the 775-kev transition, however, requires use of the γ-ray data. The γ rays from Th^{232} were first analyzed as if originating from a single state at 790 kev, and $B(E2)_d = 4.3B(E2)_{sp}$ was obtained.[7] Monopole conversion was not taken into account. This result must now be interpreted as due to the sum of γ-ray contributions from both the 775- and 788-kev states. To split off the part due only to the 775-kev state, $B(E2; 788-0)$ as determined from the conversion electron measurements was subtracted from the above value leaving a net

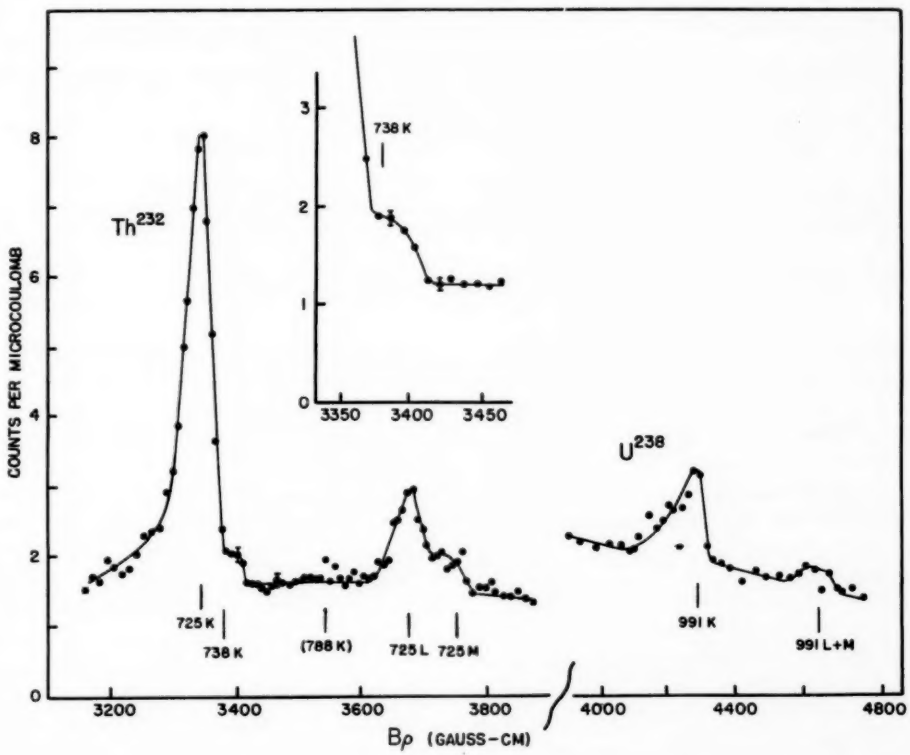

FIG. 1. The internal conversion electron spectra from Th^{232} and U^{238} targets bombarded with 5-Mev protons are shown. Self-supporting foils of natural Th (5 mg/cm^2) and natural U (57 mg/cm^2) were used. The inset shows a rerun of the region of the K-conversion lines of the weak 738-kev transition in Th^{232}, with improved counting statistics.

Table I. Experimental results and derived quantities relating to $E0$ enhancement in Th^{232} and U^{238}.

Isotope	Th^{232}				U^{238}
	Method A		Method B		
$E_{2^{+\prime}}$ (kev)	775	788	775	788	1036
$\frac{\epsilon(e^-)B(E2)_d}{B(E2)_{sp}}$	0.90[a]	0.038[a]	0.90[a]	...	0.20[a]
$\frac{\epsilon(\gamma)B(E2)_d}{B(E2)_{sp}}$	0.5[b]	3.8	2.2[c]	2.1[c]	2.4
$\frac{B(E2;2^{+\prime}\to 2^+)}{B(E2;2^{+\prime}\to 0)}$	$\frac{10}{7}$[d]	3.5^{+4}_{-2}	$\frac{10}{7}$[d]	2.6 ± 1.1	$\frac{10}{7}$[d]
$\frac{B(E2)_d}{B(E2)_{sp}}$	1.4	3.8	2.8(3.1)[e]	2.1	2.6
α	4.9	$E2$	1.1	$E2$	0.16[f]
$\mu(2^{+\prime};2^{+\prime}\to 2^+)$	4.9	0[g]	1.1	0[g]	0.15
$W(E0,e^-;2^{+\prime}\to 2^+)$	2.0×10^{11}	0	1.0×10^{11}	0	5.8×10^{10}
Ω	3.4×10^{12}	...	3.4×10^{12}	...	6.15×10^{12}
ρ	0.25	0	0.17	0	0.097

[a]Here $\epsilon(e^-)$ refers to conversion in the $2^{+\prime}\to 2^+$ transition only.

[b]This value is obtained by subtracting $\epsilon(\gamma)B(E2;788\to 0)$ from the value, 4.3, reported by Stelson and McGowan for the sum of the γ-ray partial $B(E2)$ values.

[c]For Th^{232}, $\epsilon(\gamma)$ includes all the γ rays from the $2^{+\prime}$ state.

[d]This ratio is the theoretical value.

[e]The value 2.8 was obtained by Stelson and McGowan who used the electron data of E. M. Bernstein (private communication); use of the present data gives 3.1, which was used to determine $\mu(2^{+\prime};2^{+\prime}\to 2^+)$.

[f]The γ rays from the $2^{+\prime}\to 4^+$ transition in U^{238} were not included in $\epsilon(\gamma)B(E2)$, and hence were not included in the derivation of $\alpha_{2^{+\prime}\to 2^+}$ for U^{238}.

[g]$E2$ character assumed.

for $\epsilon(\gamma)B(E2;\ 775\to 0)=0.5B(E2)_{sp}$. This value together with $\epsilon(e^-)B(E2)_d=0.9B(E2)_{sp}$ yields $B(E2;\ 775\to 0)=1.4B(E2)_{sp}$. These results are summarized in Table I under the heading "Method A," and represent the maximum utilization of the conversion electron data.

More recently Stelson and McGowan[4] also have taken into account the presence of the 775-kev state in analyzing the γ-ray spectrum of Th^{232}. While the 725-, 738-, 775-, and 788-kev γ rays are not resolved in their measurements, they have decomposed the composite peak by making use of the known intensity of the 612-kev transition to the 4^+ state from the 775-kev state, together with the theoretical ratios

$$B(E2;\ 2^{+\prime}\to 0):B(E2;\ 2^{+\prime}\to 2^+):B(E2;\ 2^{+\prime}\to 4^+) = 1:\tfrac{10}{7}:\tfrac{10}{7}, \qquad (1)$$

which they assume to be applicable.[1] By this means they obtain $\epsilon(\gamma)B(E2)_d$ for the 775- and 788-kev states separately, and finally $B(E2;788\to 0)$ and $B(E2;\ 788\to 50)/B(E2;\ 788\to 0)$. This information is summarized under the heading "Method B" in Table I.

These results have been used to evaluate the quantities of theoretical interest. The internal conversion coefficient α for the $E0$-enhanced $2^{+\prime}\to 2^+$ transitions is obtained by noting that

$$\frac{\epsilon(e^-)B(E2)_d}{\epsilon(\gamma)B(E2)_d}=\alpha\left[1+\frac{W(E2,\gamma;2^{+\prime}\to 0)}{W(E2,\gamma;2^{+\prime}\to 2^+)}+\frac{W(E2,\gamma;2^{+\prime}\to 4^+)}{W(E2,\gamma;2^{+\prime}\to 2^+)}\right]^{-1}. \qquad (2)$$

The transition probabilities $W(E2,\gamma)$ are known only for the $2^{+\prime}-0$ transitions; therefore, it is necessary to use the theoretical ratio (1) to determine the relative transition probabilities for use in (2). The relation

$$\mu(2^{+\prime};2^{+\prime}-2^{+})\equiv\frac{W(E0,e^{-};2^{+\prime}-2^{+})}{W(E2,\gamma;2^{+\prime}-2^{+})}=\alpha-\alpha_{2},$$

where α_2 is the theoretical $E2$ conversion coefficient, gives the desired ratio of transition probabilities. The monopole transition probability $W(E0,e^{-};2^{+\prime}-2^{+})$ was determined in turn from (1) and the experimental $B(E2;2^{+\prime}-0)_d$ values. The nuclear monopole strength parameter, ρ, is defined by

$$W(E0,e^{-})=\Omega\rho^{2},$$

where Ω, the electronic factor, is given graphically.[2]

Reiner[3] has treated the problem of monopole enhancement for deformed nuclei and has derived a formula for the ratio of the $E0$ to $E2$ transition probabilities from β-vibrational states. For Th^{232} his formula leads to $\mu(2^{+\prime};2^{+\prime}-2^{+})=3.8$, a value which is intermediate between the experimental values obtained by Methods A and B. For U^{238}, however, Reiner's formula gives $\mu(2^{+\prime};2^{+\prime}-2^{+})=1.4$ which is a factor of ten larger than the value obtained experimentally. The explanation of the discrepancy is not apparent in terms of the simple vibrational model used by Reiner, since Th^{232} and U^{238} have similar atomic numbers, equilibrium deformations, and vibrational energies which are the only parameters entering into the calculation. The values obtained for the nuclear strength parameter ρ are close to the approximate value of $\frac{1}{5}$ predicted by Church and Weneser for $0^{+}-0^{+}$ transitions from vibrational states in spheroidal nuclei.[2] Again, however, it is not clear why the value of ρ should differ by a factor of two between Th^{232} and U^{238}. Perhaps one sees in these discrepancies the influence of the different ground-state configurations.

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NONLOCAL POTENTIAL AND ALPHA-DECAY BARRIER PENETRABILITY

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Since the early works on the alpha disintegration problem, the barrier for the alpha emission has traditionally been taken to be purely Coulombian with an abrupt cutoff at the nuclear boundary defined by some appropriate $A^{1/3}$ law. Alpha decay was first considered[1] by the present author by assuming that the potential barrier is not purely Coulombian and should be taken as $2(Z-2)e^{2}/r-V$, where V is defined as the short-range interaction between the emitted alpha particle and the residual nucleus. Subsequently, the form of the interaction, V, which is superposed on the Coulomb field, has been taken differently by different authors[2-5] in their calculations of the barrier penetrability.

It is, however, worth while to note that in the above calculations of the penetrability factor, the nuclear part of the barrier has been taken to be purely static, while the careful analyses of the recent experimental findings strongly suggest[6-9] the velocity dependence of both the nucleon-nucleus and the alpha-nucleus interactions. On the other hand, owing to the uncertainties in the knowledge of the effect of the intranuclear dynamics

on the half-lives of the alpha-emitting elements, it is important that the calculation of the penetrability factor be made with reasonable certainty by taking all such points into consideration.

The purpose of the present work is therefore to study the effects of the velocity-dependent character of the alpha-nucleus interaction on the magnitude of the alpha-decay barrier penetrability factor. For this purpose we have taken an interaction kernel possessing a nonlocal part represented by a δ function approximated by a Gaussian exponential, so that

$$V(\vec{r},\vec{r}') = -V_0 f\left(\left|\frac{\vec{r}+\vec{r}'}{2}\right|\right)\delta_b(\vec{r}-\vec{r}'), \qquad (1)$$

and

$$\delta_b = \pi^{-3/2} b^{-3} \exp\left[-\left(\frac{r-r'}{b}\right)^2\right], \qquad (1.1)$$

where V_0 and the form function, $f(r)$, of the static part are taken from Igo's potential, viz., $V_0 = 1100$ Mev and $f(r) = \exp[-(r - 1.17A^{1/3})/0.574]$, r being in fermis; and the extent of the nonlocality, b, is given by Frahn[10] for nucleons as $b = 0.902\times10^{-13}$ cm, which is also assumed by us to be the range of nonlocality of nuclear potential for α particles. Now using the above potential in the integro-differential equation analogous to that taken by Frahn and Lemmer,[11] we find the radial part of the Schrödinger equation for the emitted alpha particle to be

$$d^2u_l/dr^2 + (2\mu/\hbar^2)[\epsilon(r)E - (\hbar^2/2\mu)l(l+1)/r^2 - 2(Z-2)e^2\epsilon(r)/r + V_0 f(r)\epsilon(r)]u_l + \eta\epsilon(r)\{f''(r)/4 - f'/2r + f'u_l'/u_l\} = 0, \qquad (2)$$

where

$$\epsilon(r) = [1+\eta f(r)]^{-1} \text{ and } \eta = (\mu b^2/2\hbar^2)V_0. \qquad (3)$$

Now in connection with the studies of nuclear energies, Green[12] has shown that the terms corresponding to the part within second braces in (2) contribute negligible shift of the eigenvalues. Therefore, neglecting in Eq. (2) the terms involving derivatives of $f(r)$, and using the WKB method of solution for simplicity, we find that the penetrability factor for the ground state transitions is given by

$$\exp\left\{-2\left(\frac{(2\mu)^{1/2}}{\hbar}\right)\int_{r_i}^{r_0}\left[\frac{2(Z-2)e^2\epsilon(r)}{r} - V_0 f(r)\epsilon(r) - E\epsilon(r)\right]^{1/2} dr\right\}. \qquad (4)$$

The integral in the exponential has been calculated numerically by using Simpson's rule and taking 120 strips. The turning points r_i and r_0 remain unaffected as for a static potential since the modifying factor $\epsilon(r)$ in the integral is common to all the terms.

It is interesting to note that the effect of the nonlocal interaction is quite remarkable. For example, in the case of $_{100}Fm^{254}$, calculation gives a value for the penetrability factor of 5.848×10^{-24}, which is about double the value obtained with the corresponding static potential, viz., 3.524×10^{-24}.

Numerical calculations for the whole series of the alpha-active elements and for their excited-state transitions are in progress and a detailed report will be published in due course.

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CHARGED PARTICLE AND TOTAL REACTION CROSS SECTIONS FOR PROTONS AT 9.85 Mev*

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The optical model can predict elastic scattering, polarization, and total reaction cross sections. Recent parameter studies[1] indicate that at least a necessary condition for obtaining a unique set of parameters is the existence of data covering all three quantities at the same energy. Until now there has been no energy (or bombarding particle) for which such complete data are available.

Near 10 Mev, both differential elastic[2,3] and polarization measurements[4] have been made for many nuclei with protons, and thus reaction cross sections are particularly desirable. In addition, there are reasons to believe[2] that it is just at charged particle energies in the vicinity of the Coulomb barrier that reaction cross sections will be most useful in resolving the ambiguities in the optical model parameters. This note reports measurements of the sum of the cross sections for all reactions in which a charged particle is emitted. This quantity, which we designate $\sigma(p,q)$, when added to the total (p,n) cross section gives essentially the total reaction cross section, σ_R. The values of σ_R which we obtain are shown to disagree with current optical model predictions. The significance of this disagreement is discussed.

Direct attenuation measurements have been made of σ_R with protons of 34 and 61 Mev.[5,6] These methods become progressively more difficult as the proton energy is reduced. At 10 Mev, however, it is feasible to measure all of the significant partial reaction cross sections, i.e., $\sigma(p,n)$, $\sigma(p,p')$, $\sigma(p,\alpha)$, etc., and sum to obtain σ_R. Reactions in which two or more particles are emitted are either energetically forbidden or greatly suppressed because of the Coulomb barrier. The experiment reported here was designed to measure the energy and angular distribution of all charged particles emitted from the nucleus down to the lowest energy allowed by the Coulomb barrier. Separate measurements are then needed of the (p,n) cross section to obtain σ_R. The experiment was performed using 9.89-Mev protons from the Minnesota linac. A CsI detector and pulse-height analysis were used to obtain complete charged particle spectra at 10° intervals from 20° to 150° for a number of elements. Figure 1 shows a spectrum for Zn^{66} at 150°. The shaded region shows the background, taken with an absorber over the detector to stop all charged particles.

The spectra were integrated over energy (leaving out the elastic peak) and angle to obtain $\sigma(p,q)$ for each element. The results are given in Table I. The angular distributions of the nonelastic charged particles were found to be isotropic except for a slight forward peaking due to a small direct component in the inelastic protons to the first excited state of the target nucleus. A complete presentation of the spectra together with an analysis of the energy and angular distributions will appear in a later paper.

It can be seen from Table I that there is a considerable variation in the values of $\sigma(p,q)$ from element to element. This is to be expected from a statistical theory of the decay in which the ratio of inelastic scattering to neutron emission is strongly dependent on the $(p,n)Q$ value and on the Coulomb barrier height.

To obtain σ_R, the values of $\sigma(p,n)$, $\sigma(p,\gamma)$, and σ_{ce} (compound elastic) must be added to $\sigma(p,q)$. Measurements of $\sigma(p,\gamma)$ near 10 Mev give values of 1-2 mb,[7] so this quantity can be neglected. Compound elastic cross sections for protons have not yet been measured. An upper limit to σ_{ce} would be approximately the isotropic part

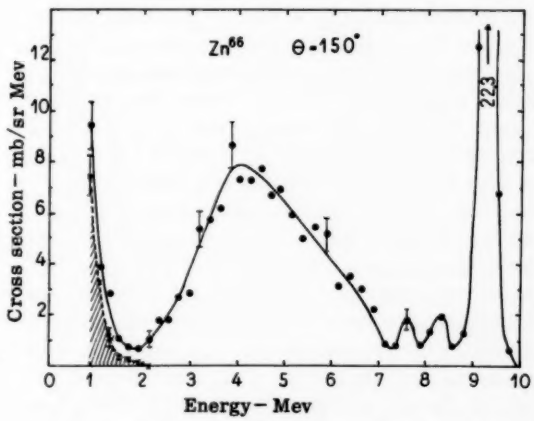

FIG. 1. Charged particle spectrum for Zn^{66} at 150°. Energies are in the laboratory system.

Table I. Charged particle and total reaction cross sections at 9.85 Mev.

Target[a]	$\sigma(p,q)$ mb	$\sigma(p,n)$ mb	σ_R, experimental mb	σ_R, optical model mb
C	$>228 \pm 15$[b]			
Al	725^{+60}_{-30}	>16[c]	$>741^{+60}_{-30}$	606
Ti	362 ± 25	$>371 \pm 30$[d]	$>733 \pm 40$	
V	$>165 \pm 10$[e]	552^{+170}_{-120}	$>717^{+170}_{-120}$	
Fe	498 ± 33	$>367 \pm 40$[f]	$>865 \pm 52$	
Co	366 ± 27			
Ni	590 ± 30	>123[g]	$>713 \pm 40$	576
Cu^{63}	365 ± 18	480^{+90}_{-65}	845^{+92}_{-67}	
Cu^{65}	155 ± 10	819 ± 75	974 ± 76	670 to 800[h]
Zn^{66}	306 ± 21	640 ± 80[i]	946 ± 85	
Ag	30.7^{+3}_{-7}			
Sn	28 ± 6			425

[a] The targets were of natural isotopic composition except for Cu and Zn.

[b] Some of the low-energy alphas from the $(p,3\alpha)$ reaction were below the detection threshold.

[c] Value of $\sigma(p,n)$ at 6.7 Mev, reference 10.

[d] Does not include $\sigma(p,n)$ for $Ti^{46,47,48}$ (20.5% relative abundance).

[e] Does not include $\sigma(p,p')$ to states in V^{51} at 0.32 and 0.65 Mev which could not be resolved.

[f] Does not include $\sigma(p,n)$ for Fe^{54} (5.8% relative abundance).

[g] Does not include $Ni^{58}(p,n)Cu^{58}$.

[h] Range of values for Cu for $r_0 = 1.20$ to $r_0 = 1.33$, references 1 and 16.

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of the cross section for exciting the first state of the nucleus. For the medium weight even-even nuclei this was found to be ~50 mb. For the odd-A nuclei σ_{ce} is expected to be considerably less.[8]

Unfortunately only a few measurements of $\sigma(p,n)$ exist near 10 Mev,[9-13] and these measurements are not in good agreement with each other. The values of $\sigma(p,n)$ shown in Table I are more or less arbitrarily weighted averages of the various values. For the natural isotopic targets the $\sigma(p,n)$ value quoted is the weighted sum of the cross sections for those isotopes for which measurements exist. The errors indicated cover the disagreements between the several experiments. The column marked "σ_R, experimental" is the sum of our $\sigma(p,q)$ value and the value chosen for $\sigma(p,n)$. Only lower limits to σ_R can be given when (p,n) measurements are not available for all of the isotopes of the target. No attempt has been made to estimate σ_{ce}, and thus the σ_R values must be considered lower limits to the total reaction cross sections as computed in the optical model. It is felt that the large difference in $\sigma_{R,\,exp}$ for the two copper isotopes may not be real but may be due to the large errors in the (p,n) measurements. For comparison with the optical model, the average for the two copper isotopes, $\bar{\sigma}_R = 910 \pm 60$ mb, should be used, along with the value of $\sigma_R = 946 \pm 85$ mb for Zn^{66}. In the optical model, the value of σ_R for Zn^{66} is expected to be within a few percent of those for the copper isotopes. In the schematic theory of compound nucleus formation,[14] in which the nucleus is depicted as a uniform, completely absorbing sphere, a radius constant of $r_0 = 1.75$ is needed to give $\sigma_R = 946$ mb for Zn^{66}. This is a considerably larger value than has been used in fitting (p,n), (α,n), etc., excitation func-

tions,[14] but in these cases a significant part of the reaction cross section has been omitted, namely, the inelastic scattering.[15]

The relevant optical model values of σ_R are those of Glassgold et al.[16] using a central potential and those of Nodvik and Saxon[1] calculated with the inclusion of a spin-orbit term. Nodvik and Saxon obtain for copper, after fitting the elastic scattering and polarization data, a value of $\sigma_R = 670$ mb for $r_0 = 1.20$ and $\sigma_R = 800$ mb for $r_0 = 1.33$. The best fit to the elastic and polarization data is obtained for $r_0 = 1.25$, for which $\sigma_R = 720$ mb. The values of σ_R obtained by Glassgold, without the spin-orbit term, are very close to the above. In both cases calculations were made with the same Saxon form factor for the real and imaginary parts of the potential, $V(r)$ and $W(r)$.

It is seen that the optical model values are from 100 mb to 200 mb lower than the experimental values for iron, copper, and zinc and in addition the $\sigma(p,q)$ values alone for Ni and Al are comparable to or greater than the optical model values for σ_R. The inclusion of σ_{ce} in the experimental σ_R would further increase these discrepancies.

Preliminary calculations by Easlea[17] show that by putting all of the absorption at the nuclear surface the reaction cross section can be raised to 844 mb for copper with $r_0 = 1.30$, still fitting the elastic and polarization data.

We feel that this value of σ_R is still too small and that our data constitute strong evidence for the extension of $W(r)$ beyond $V(r)$ as has been predicted theoretically.[18] More calculations as well as more accurate (p,n) cross-section measurements are needed to settle this question.

Finally it should be noted that either a surface peaking or an extension of $W(r)$ to a larger radius will lower the central value of W to one more in line with predictions of the Brueckner theory for infinite nuclear matter.[17]

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INVESTIGATION OF MEDIUM AND HEAVY NUCLEI BY QUASI-FREE PROTON-PROTON SCATTERING*

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Recently, experimental groups in Uppsala,[1] Harwell,[2] and Chicago[3] have directly demonstrated the shell structure of several light nuclei by measuring the energy and angular correlations of proton pairs resulting from high-energy (150-440 Mev) quasi-free knockout processes. The binding energies and interesting features of the momentum distributions of the protons in the $1s$ and $1p$ shells and the spin-orbit splitting of the $1p$ shell have been determined in this way.

Up to now these measurements have been limited to light nuclei, mainly because the cross section for knockout processes in the inner shells and the level distances in the upper shell decrease rapidly with increasing nuclear radius.[4] It is the purpose of this note to point out that high-energy quasi-free p-p scattering is not only able to supply new information on the light nuclei, but is also a promising tool for the investigation of medium and heavy nuclei. It is expected that the limitation in size of the effective interaction zone, resulting from the absorption of the protons by the nucleus, should give rise to a typical angular correlation pattern from which several important quantities may be read off directly.

Throughout this paper the indices 0, 3, 1, 2, i, and f refer, respectively, to the incoming and the nuclear proton before the collision, to the two outgoing protons, and to the initial and final nucleus. The symbol $\vec{k}$ refers to momentum; α_i is the angle which $\vec{k}_i$ makes with the projection of $\vec{k}_0$ on the plane through $\vec{k}_1$ and $\vec{k}_2$ (see Fig. 1); m denotes a magnetic quantum number and $\vec{P}$ the parity operator.

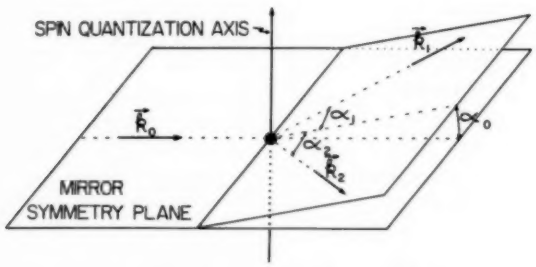

FIG. 1. Definition of angles and plane of mirror symmetry.

Consider as an example the coplanar ($\alpha_0 = 0$) quasi-free scattering of 150-Mev protons by medium and heavy nuclei and choose α_1 and α_2 not too different from 45°. Comparing the mean free paths in nuclear matter of the involved protons with the total length of their trajectories in the nucleus, one sees that the bulk of the protons which suffer only one collision originates from the two pole caps at the ends of the nuclear symmetry axis orthogonal to the scattering plane.[5,6] The contribution of the multiply scattered proton pairs to the correlation cross section is smeared out over the whole available energy and angular range. It results in a smooth background in the measured energy spectrum on which the peaks caused by the relevant clean knockout collisions are superimposed. It is to be expected that the height of the aforementioned pole caps is small compared with their mutual distance D. The contributions of the two pole caps will, because of symmetry reasons, be equal in magnitude, though they have in general a phase difference η. If one changes α_0 to small nonzero values (keeping α_1, α_2, k_1, k_2 constant), the magnitudes of the pole cap contributions remain approximately equal, but their phase difference becomes $\eta + \alpha_0 k_0 D$. For the transition to a definite final state of the nucleus, the contributions of the two pole caps will add coherently and the correlation cross section will show an interference pattern of the form

$$|G(\alpha_0, \alpha_1, \alpha_2, k_1, k_2)|^2 \cos^2[\tfrac{1}{2}(\alpha_0 k_0 D + \eta)]. \quad (1)$$

As a function of α_0, $|G|^2$ varies slowly as compared to the $\cos^2$ term (which in our example has a period of about 10°).

The value of η can be found by a symmetry argument. Consider the operator Q_z which mirrors the wave function through the plane $\alpha_0 = 0$. We quantize the spins in the direction orthogonal to this plane and note that Q_z equals the parity operator times a rotation by an angle π around the quantization axis. The initial state is an eigenstate of Q_z with eigenvalue $P_i \exp[i(m_i + m_0)\pi]$. The mirror parity operator of the final state is $P_f \exp[i(m_f + m_1 + m_2)\pi] Q_{\alpha_0}$, where Q_{α_0} is the mirror parity of the interference pattern. Parity

conservation results in the condition:

$$Q_{\alpha_0} = P_i P_f \exp[i\Delta m\pi], \qquad (2)$$

with

$$\Delta m = (m_i + m_0) - (m_f + m_1 + m_2).$$

Because of fermion conservation, Δm is an integer. Thus, only the cases $Q_{\alpha_0} = \pm 1$ occur.

Using the single-particle model and the impulse approximation (for which, by the same symmetry arguments, $m_0 + m_3 - m_1 - m_2 = 0$), one obtains $Q_{\alpha_0} = (-1)^{l+m}$, where l and m refer to the Y_l^m component of the state from which the proton has been knocked out.

From (1) and (2) it follows, independent of the experimental energy resolution, that the correlation cross section will have the form

$$a(\alpha_0, \alpha_1, \alpha_2, k_1, k_2)\cos^2(\tfrac{1}{2}\alpha_0 k_0 D) + b(\alpha_0, \alpha_1, \alpha_2, k_1, k_2)\sin^2(\tfrac{1}{2}\alpha_0 k_0 D), \qquad (3)$$

where a and b are slowly varying functions of α_0.

If a measurement is made in such a way that many final states of different character contribute statistically to the cross section, a and b will have similar magnitudes and the α_0-dependent interference pattern will fade out. On the basis of the shell model and of the nature of the reaction, one expects to be able to resolve groups of states which correspond to the shell-model state from which the proton has been ejected. But, because the m degeneracy cannot be resolved, one in general will still have a mixture of $\cos^2$ and $\sin^2$ contributions.

For this last case, detailed calculations[6] have been made with a single-particle model, the impulse approximation, and a simplified distortion for the incoming and outgoing waves. After summing over the magnetic quantum numbers, the coefficients a and b are shown to be strongly different for small momentum transfers (nonrelativistically: $\alpha_1 \approx \alpha_2 \approx 45°$; $k_1 \approx k_2 \approx \frac{1}{2}\sqrt{2}\,k_0$). The expected α_0-dependent interference pattern is therefore pronounced and gives by its period the distance between the pole caps, by its modulation the height of the pole caps, and by its phase the parity of the shell-model state from which the proton has been knocked out. Furthermore, it turns out that the orbital angular momentum of this state can be found from the (α_1, α_2) dependence of the correlation cross section.

Finally, we want to point out two necessary conditions for the applicability of our general treatment. At a given bombarding energy the nucleus to be investigated must be at least so heavy that the heights of the contributing pole caps are rather small compared to the nuclear radius. On the other hand, these heights should not be too small, because otherwise the absolute magnitudes of the pole cap contributions become already very different for small values of α_0 and the correlation pattern fades out. A more quantitative prediction requires distorted wave calculation for each case.

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PION CLOUD EFFECTS AND THE TWO-CENTER MODEL OF COSMIC-RAY JETS

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(Received July 12, 1960)

Recent investigations of high-energy nuclear interactions[1-6] have shown that at energies above 1 Gev an important part is played by peripheral collisions involving only one or two pions in the meson cloud of the colliding nucleons. In the present note, an attempt is made to interpret the two-cone structure of cosmic-ray jets along these lines.

Indeed, it has been shown[7,8] that most jets can be interpreted as two bunches of particles emerging from two centers moving in opposite directions each with a Lorentz factor $\bar{\gamma}$ in the center-of-mass system of the colliding nucleons. Until now, $\bar{\gamma}$ has been treated as a free parameter, supposed to be related in some way or other to the impact parameter and providing essentially an estimate of the inelasticity of the collision. Perhaps the most remarkable fact of the results obtained until now by means of the two-center model is the cluster of values of $\bar{\gamma}$ about a mean value ~1.5 - 1.6.

On the other hand, the pion-cloud effects detected at lower energy ought to appear even more pregnantly at the high energies of the cosmic-ray jets, owing to the very short deBroglie wavelengths of the colliding particles. This means that at reasonable impact parameters, each of the two nucleons has a large chance to collide with one single pion in the meson cloud of its collision partner. The process is then essentially a pair of head-on pion-nucleon collisions.[9,10] Owing to the fact that the incoming nucleon and its pion cloud move with the same velocity in the lab system (Lorentz factor γ_0), the kinematics of the two quasi-independent pion-nucleon collisions automatically lead to a fixed value of $\bar{\gamma}$ at high γ_0 values (say, above 10^{11} ev).

Let γ_1 be the Lorentz factor for the collision of the incoming nucleon with the pion at rest in the lab system, and γ_2 the corresponding quantity for the collision of the incoming pion with the target nucleon. We have then (see reference 10)

$$\gamma_1 = (\gamma_0 + \mu)(1 + \mu^2 + 2\mu\gamma_0)^{-1/2}, \quad (1)$$

$$\gamma_2 = (\mu\gamma_0 + 1)(1 + \mu^2 + 2\mu\gamma_0)^{-1/2}, \quad (2)$$

where μ is the pion rest mass ($M = c = 1$). For $\gamma_0 \gg 1$ these expressions reduce to

$$\gamma_1 \approx (\gamma_0/2\mu)^{1/2}, \quad (3)$$

$$\gamma_2 \approx (\mu\gamma_0/2)^{1/2}. \quad (4)$$

Therefore, obviously,

$$\gamma_1\gamma_2 \approx \gamma_0/2 \approx \gamma_c^2, \quad (5)$$

where γ_c is the Lorentz factor of the c.m. system of the colliding nucleons with respect to the laboratory frame,

$$\gamma_1/\gamma_2 = (\gamma_0 + \mu)/(\mu\gamma_0 + 1) \approx \mu^{-1}, \text{ (for large } \gamma_0) \quad (6)$$

Table I. Comparison of computed and observed quantities for the two-center model. All experimental γ values are Castagnoli estimates[a]; for $\gamma_c \geq 7$, they have to be corrected by a factor[b,c] $f \sim 0.7$. At lower γ_c values this correction factor approaches unity.[b] Since $\bar{\gamma}$ varies as $\gamma_c^{1/2}$, the corresponding correction factor has been taken as $\sim f^{1/2}$. In the range $5 < \gamma_c \leq 10$ the interpolation formula deduced in reference 1 has been used. The expected values are weighted averages.

Energy range	$\langle\gamma_c\rangle$	$\bar{\gamma}$ Computed	$\bar{\gamma}$ Observed	$\bar{\gamma}$ Corrected	γ_1/γ_2 Computed	γ_1/γ_2 Observed	γ_1/γ_2 Corrected
$1 < \gamma_c \leq 5$	3.7	1.37	1.35 ± 0.05	...	5.20	5.15 ± 0.58	...
$5 < \gamma_c \leq 10$	7.0	1.43	1.56 ± 0.06	1.39 ± 0.05	6.45	8.00 ± 1.18	6.40 ± 0.95
$\gamma_c > 10$	44.3	1.49	1.89 ± 0.13	1.59 ± 0.11	6.67	13.70 ± 2.42	9.56 ± 1.69
All jets	19.3	1.49	1.63 ± 0.06	1.47 ± 0.05	6.67	9.45 ± 0.06	6.61 ± 0.75

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and

$$\bar{\gamma} = (\mu+1)\gamma_c(1+\mu^2+2\mu\gamma_0)^{-1/2} \approx \tfrac{1}{2}(\mu^{1/2}+\mu^{-1/2}), \quad (7)$$

in the same approximation as above.

Hence, at large energies we would expect

$$\gamma_1/\gamma_2 \approx 6.67, \quad (8)$$

and

$$\bar{\gamma} \approx 1.49. \quad (9)$$

At low energies (say, $\gamma_c < 7$ or $\gamma_0 < 100$), both γ_1/γ_2 and $\bar{\gamma}$ rapidly decrease and approach unity.

According to the above ideas, an analysis has been made of 78 jets (p, n, and α events)[11] measured in the Prague Laboratory. The results are shown in Table I. The agreement between the expected and the observed values and their trends is obvious.

Further applications of this method may be thought of for the case of nucleon-nucleus and nucleus-nucleus collisions, where peripheral interactions are expected to be sometimes superimposed on head-on nucleon-nucleon collisions.

The author is very indebted to Dr. J. Pernegr, who kindly put at his disposal detailed data of the jets recorded and measured in the Prague part of Po-Stack No.1.

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PROOF OF THE MANDELSTAM REPRESENTATION IN PERTURBATION THEORY*

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(Received August 9, 1960)

I wish to report a proof that the Mandelstam representation[1] for a scattering amplitude is valid for every term in the perturbation series for the amplitude. The proof applies to any system of interacting particles that does not have anomalous thresholds. The latter requirement can be expressed by the mass conditions obtained in fourth order. In this Letter I will outline the main steps of the proof, using the equal-mass case as an illustration. The details are contained in two papers[2,3] in the course of publication and a third[4] shortly to be submitted.

The invariant energies squared will be denoted s, t, and u; a term in the expansion of the amplitude $A(s,t)$ will be written $F(s,t)$. The main steps include the following:

(1) The physical branch of F in a physical scattering region has the representation

$$F(s,t) = \lim_{\epsilon\to 0} c\int_0^1 d\alpha_1\cdots d\alpha_n \frac{\delta(1-\sum\alpha_i)[C(\alpha)]^{n-2l-1}}{[D_\epsilon(\alpha,s,t)]^{n-2l}},$$

where we have (I, Sec. 6),[2]

$$D_\epsilon(\alpha,s,t) = sf(\alpha)+tg(\alpha)-m^2K(\alpha)+i\epsilon C(\alpha)\textstyle\sum\alpha_i$$
$$= D(\alpha,s,t)+i\epsilon C(\alpha)\textstyle\sum\alpha_i.$$

Singularities (i.e., branch points) of F will occur when D_ϵ has end-point zeros or coincident zeros (pinching the integration contour) in each α variable as ϵ approaches 0. End-point zeros can be reinterpreted by using reduced diagrams.

(2) $D(\alpha,s,t)$ is negative for real positive α in the region $s>0$, $t>0$, $u>0$. Hence F is real in this region [I, Secs. 4 and 8 (F)].

(3) The only straight lines of singularities of the physical branch of F are normal thresholds, since they must intersect a physical scattering region (I, Sec. 5).

(4) Curves of singularities $\Gamma(s,t)$ of F in the real s,t plane have slope (I, Sec. 7)

$$dt/ds = -f(\alpha)/g(\alpha),$$

and they have normal thresholds as asymptotes.

(5) From (2), (3), and (4) it can be shown (III)[4]

that there are no singularities in the region $s<4m^2$, $t<4m^2$, $u<4m^2$, and that D is negative and F real in this region. It also follows that no curve of singularities enters the real s,t plane in the region $-4m^2<t<4m^2$.

(6) A single-variable dispersion relation in s can be proved (III) for t real and with $(-4m^2<t<4m^2)$, by noting that for $s>4m^2$ and real α we have

$$D(\alpha, s+i\epsilon, t)\neq 0.$$

We obtain the correct physical branch of $F(s,t)$ in the physical region $s>4m^2$ by defining it from

$$F(s,t) = \lim_{\epsilon\to 0} c\int_0^1 d\alpha_1\cdots d\alpha_n \frac{\delta(1-\sum\alpha_i)[C(\alpha)]^{n-2l-1}}{[D(\alpha, s+i\epsilon, t)]^{n-2l}}.$$

No distortion of the real α contours is necessary except in the limit as ϵ approaches 0. This proves that $F(s,t)$ has no singularities in the upper half s plane when t is real and in $(-4m^2<t<4m^2)$. It also defines the physical sheet of the complex variable s.

(7) If a diagram, or reduced diagram, has its first thresholds at $s=a$, $t=b$, $u=c$, the limits $\pm 4m^2$ in (5) and (6) can be extended to suitable combinations of a, b, and c. The discussion below can then be carried through by using these limits.

(8) The curves of singularities $\Gamma(s,t)$ defined by $t=t(s)$, with s and t real, will lie on surfaces of singularities $\Sigma(s,t)$. These will be singularities of some analytic continuation of F. Our problem is to prove that none of these surfaces correspond to singularities on the physical sheets of s, t, and u. This can be done by proving that (a) in the region $s>4m^2$, $t>4m^2$, all curves of singularities have negative slope, and (b) there are no disconnected complex singularities in the physical sheet (II).[3]

(9) There are two alternative methods which give these results. I will illustrate the first (III) by examining the continuation of $F(s,t)$ past $t=4m^2$ along any path in the upper half of the s plane and with $t=t_1+i\epsilon$. A path of this type leads to the physical branch of F in a physical region, and it lies on the physical sheet. If no such path meets a singularity of F, our required result is true. If the path does meet a singularity, it may be either a disconnected complex singularity or a complex singularity coming from a spurious turning point. By continuity, the latter must occur at a minimum of t along a curve of singularities, since these curves cannot disappear from the physical sheet except asymptotically through normal thresholds. If any path meets either type of singularity, choose the singularity on the physical sheet for which t_1 is nearest to $4m^2$, if possible. But we can then trace the singularity along its related singular surface Σ on the path $s_1+is_2 = s(t_1+i\epsilon)$ to smaller values of t_1 until t_1 is less than $4m^2$. This path does not meet either $t=4m^2$ or $s=4m^2$, since for finite t_1 the function $F(s_1+is_2, t_1+i\epsilon)$ is analytic near these thresholds. Letting ϵ tend to zero, we obtain a contradiction of our single-variable dispersion relation below $t=4m^2$. This proves our required result that there are no spurious turning points and no disconnected complex singularities.

(10) The second method (III) uses the elegant device of analytic completion.[5] The result of Sec. (6) is first extended to show that $F(s,t_1\pm i\epsilon)$ is analytic for s in the upper half plane. Then we define, for $t_1<4m^2$,

$$F(s,t_1+i\epsilon)=\frac{1}{2\pi i}\int_C \frac{F(z,t_1+i\epsilon)dz}{z-s},$$

where C is a semicircle in the upper half plane. The contour C can now be displaced in the complex s,t space by continuously increasing t_1. Provided the contour does not meet any singularities, this procedure defines an analytic continuation in the upper half s plane for all real positive values of t_1. There are no anomalous thresholds, and the contour is not distorted near the normal thresholds since near these $F(s_1+is_2, t_1+i\epsilon)$ is analytic. Spurious turning points (minima in t) and disconnected complex singularities cannot exist, since they would lead to horns extending into the volume swept out by C. This proves that the analytic completion we require is possible. The curves on which $F(s_1+i\epsilon', t_1+i\epsilon)$ is singular, as ϵ and ϵ' tend to 0, will have normal thresholds as asymptotes and will have negative slope.

(11) A similar discussion (III) shows that $F(s+i\epsilon, t-i\epsilon')$ analytically continued from the real region is not singular for $s>4m^2$, $t>4m^2$ except in the limit ϵ, $\epsilon'\to 0$. In this limit it is singular only at the normal thresholds and not on the curves of singularities. Thus where the slope of Γ is negative, it extends to Σ only at points $s+i\epsilon$, $t-i\epsilon'$ that lie in nonphysical sheets.

(12) From (10) and (11) a double application of Cauchy's theorem is possible (II) and establishes

the validity of the Mandelstam representation for every term F in the perturbation series for the amplitude A.

I am indebted to Mr. J. Nuttall and Dr. K. Symanzik for valuable discussions. I wish also to thank Dr. J. C. Polkinghorne for an outline of his work (in collaboration with Mr. P. Landshoff and Dr. J. C. Taylor) on a different approach to a proof of the Mandelstam representation.

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ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

TEMPERATURE DEPENDENCE OF THE LOW-MOMENTUM EXCITATIONS IN A BOSE GAS OF HARD SPHERES. F. Mohling, Department of Physics, Cornell University, Ithaca, New York, and M. Morita, Department of Physics, Columbia University, New York, New York (Received June 24, 1960).

The pseudopotential method is used to calculate the temperature dependence of the low-momentum excitations in a Bose gas of hard spheres to an order beyond that previously known. The correction term is shown to be small compared to the leading term for all temperatures less than the critical temperature T_c. The excitations become poorly defined for temperatures slightly below T_c, however, since it is shown that in this region the excitation lifetimes are quite small.

STATISTICAL MECHANICS FOR THE NON-IDEAL BOSE GAS. A. E. Glassgold, A. N. Kaufman, and K. M. Watson, Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California (Received May 16, 1960).

The equilibrium and quasi-equilibrium properties of a system of interacting bosons are studied from a microscopic point of view. For equilibrium, the model of Bogolyubov is generalized to finite temperature by using the grand partition function. The thermodynamic properties and the pair-correlation function are calculated. The statistical mechanics for moving systems is then developed and applied to the problem of a rotating fluid. For quasi-equilibrium, general transport equations are derived from first principles, independent of statistics and model. For the Bogolyubov model, the familiar two-fluid hydrodynamics is then derived, leading to the phenomena of first and second sound.

DIELECTRIC FORMULATION OF QUANTUM STATISTICS OF INTERACTING PARTICLES. F. Englert* and R. Brout, Laboratory of Atomic and Solid State Physics and Department of Physics, Cornell University, Ithaca, New York (Received February 25, 1960).

Formal relations between the free energy, the "dielectric constant" which expresses the reponse of the system to an external perturbation, and the two-particle Green's function in temperature space are derived. Connection with perturbation expansion for free energy and for general admittance tensors are established. These results are applied to a general discussion of the random phase approximation at finite temperature and it is shown that the sum on ring diagrams corresponds to the calculation of the dielectric constant in the approximation of neglect of local field corrections.

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STREAMER MECHANISM IN FILAMENTARY SPARK BREAKDOWN IN AIR AND ARGON BY FAST PHOTOMULTIPLIER TECHNIQUES. L. B. Loeb, G. G. Hudson,* R. G. Westberg,† and H. C. Huang,‡ Department of Physics, University of California, Berkeley, California (Received June 1, 1960).

This study was undertaken to demonstrate by direct photomultiplier observation that the transition from a positive point corona or a Townsend predischarge in more uniform field geometry to a transient filamentary arc in what is commonly called the electric spark proceeds as a sequence of luminous streamer mechanisms starting at the anode and crossing to the cathode at high speeds. Two photomultipliers with viewing slits transverse to the direction of propagation were used; one focused just below the anode served to trigger the sweep of a 517 Tektronix Oscilloscope which registered the passage of luminous pulses viewed by the second photomultiplier at various points across the gap. The observations of relative intensity plotted against time permitted the cross-plotting of distribution of luminosity across the gap at various time intervals after the breakdown started at the anode. In room air at atmospheric pressure for a wide range of gap forms the transition consisted in a fast primary streamer followed by a more intense secondary or even a tertiary streamer crossing from anode to cathode. On

arrival of the secondary or the tertiary streamer at the cathode, the luminous arc channel was established and increased in luminosity until the capacity was discharged. For highly divergent anode fields much branching of primary streamers occurs. The secondaries follow only a few of these and the crooked spark channel follows the path of the fastest and most effective secondary streamer on arrival at the cathode. These investigations were carried over to Linde's spectroscopically pure A gas in a system with Alpert vacuum techniques for pressures ranging from 300 mm down. Despite low luminosity, it was established that the filamentary spark transition from corona to a transient arc, as in air, progressed from anode to cathode by means of primary and secondary streamers down to 50 mm at which point the spark channel was diffuse with no recognizable streamers. Addition of 1% air at 60 mm of A restored the streamer sequence. Argon purified by prolonged gas cataphoresis showed no streamer mechanism at 240-mm pressure. Thus in the absence of adequate photoionization by its own photons, the transition in pure A consists in the development of a diffuse arc-like channel on a time scale of microseconds in contrast to tenths of a microsecond with streamers when photoionizable impurities are present.

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ANISOTROPIC FERMION GAS. G. Wentzel, Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois (Received June 27, 1960).

An energy term discovered by Kohn and Luttinger is shown to have a simple geometrical meaning.

THERMAL CONDUCTION IN ROTATING LIQUID HELIUM II. C. E. Chase, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts (Received June 30, 1960).

When a heat current flows in a wide channel filled with liquid helium II, the resulting temperature gradient is approximately proportional to the cube of the heat current density. The establishment of this gradient requires a time τ which is a function of heat current, temperature, and past history of the helium. The present experiments concern the effect of uniform slow rotation, about an axis normal to the direction of heat flow, upon grad T and τ. Grad T was measured at 0 and 1.3 rad/sec, the highest angular velocity at which this measurement could conveniently be carried out. No effect of rotation could be observed; however, an approximate calculation suggests that grad T might increase detectably at somewhat higher rates of rotation. τ was measured at a number of angular velocities between 0 and 4 rad/sec; it was found that τ was appreciably reduced by rotation, the effect being greatest at small heat currents and high angular velocities. These results can be explained on the assumption that mutual friction results from turbulence in the superfluid component, taking the form of a tangled mass of vortex line. The delay time τ characterizes the rate of growth of this turbulence when a heat current is switched on; rotation reduces τ by introducing an initial length of vortex line which accelerates this growth.

IRREVERSIBLE PROCESSES IN ISOLATED SYSTEMS. Edward N. Adams, IBM Research Center, Yorktown Heights, New York (Received May 10, 1960).

The point of view is taken that the irreversibility paradox of Loschmidt and Zermelo arises because there are two valid transport equations, one causal, the other anticausal, each consistent with the fundamental equations of mechanics. From this point of view the problem of irreversibility is to characterize a given nonequilibrium distribution as to which transport equation it will obey. Using the transport theories of Kohn and Luttinger and of Van Hove, we obtain a statistical criterion capable of so characterizing distribution functions of both stationary and time-varying types. We discuss the question of how experimental procedures consistently bring about a situation to which the causal transport equation applies; we answer the question for a very simple kind of experimental situation.

SPIN SUSCEPTIBILITY OF AN ELECTRON GAS. P. A. Wolff, Bell Telephone Laboratories, Murray Hill, New Jersey (Received June 24, 1960).

The generalized random phase approximation is used to investigate the effects of electron-electron interaction on the spin susceptibility

of an electron gas. Within this approximation, the induced spin density is determined by the solution of an integral equation that describes exchange scattering between virtually excited particles. Solutions to this equation are obtained in simple cases. They indicate that the exchange scattering enhances the susceptibility over that predicted by the Ruderman-Kittel formula by factors which, for electron densities comparable to those found in good metals, are in the range 1-2. These results can also be derived by summing certain classes of diagrams in many-body perturbation theory. The structure of these graphs shows that the random phase approximation takes account of the simplest sort of self-energy effect and the simplest type of scattering of virtually excited particles.

ELECTRO-OPTIC KERR EFFECT AND POLARIZATION REVERSAL IN DEUTERIUM-DOPED ROCHELLE SALT. H. H. Wieder and D. A. Collins, U. S. Naval Ordnance Laboratory, Corona, California (Received June 30, 1960).

Polarization reversal as a function of nucleation and growth of domains in ferroelectric deuterium-doped Rochelle salt was investigated by means of the electro-optic Kerr effect. The results indicate that a phenomenological model based on statistical nucleation of domains in a plane including the ferroelectric axis followed by a two-dimensional sidewise expansion of the domains, will adequately account for the experimental observations for fields larger than 50 volts/cm. For lower fields, the process is controlled primarily by the nucleation of new domains due to localized stresses which hinder the displacement of domain walls.

VACANCY-VACANCY INTERACTION IN COPPER. V. G. Weizer and L. A. Girifalco, Lewis Research Center, National Aeronautics and Space Administration, Cleveland, Ohio (Received June 16, 1960).

The binding energy of two vacancies in a static lattice as a function of their separation and the positions of their displaced neighboring atoms has been calculated using a Morse potential function model for copper. It was found that two vacancies attract one another at separation less than about 7 A. At separations greater than 7 A the vacancies do not interact appreciably. The most stable separation was found to be the first nearest neighbor separation or the divacancy configuration, for which the binding energy was found to be 0.64 ev. Based on these calculations, it is shown that third-stage annealing in irradiated copper may be accounted for by divacancy migration. The role of the divacancy in copper self-diffusion is also explained.

DE HAAS-VAN ALPHEN EFFECT IN GRAPHITE BETWEEN 3 AND 85 KILOGAUSS. W. J. Spry and P. M. Scherer, Research Laboratory, National Carbon Company, Parma, Ohio (Received June 22, 1960).

The de Haas-van Alphen effect has been measured in a single crystal of graphite between 3 and 85 kilogauss with the c axis of the crystal parallel to the magnetic field. Two periods were observed: 2.05×10^{-5} gauss^{-1} for the electronic component and 1.53×10^{-5} gauss^{-1} for the holes. No new and unexpected oscillations were observed at magnetic fields as large as 100 kilogauss.

FLUORESCENCE AND REFLECTION SPECTRA OF NaI SINGLE CRYSTALS. W. J. Van Sciver, Levinthal Electronic Products, Incorporated, Palo Alto, California (Received April 18, 1960).

The fluorescence emission and excitation, reflection, and transmission spectra of 22 different NaI crystal samples, grown in crucibles of Pt or quartz under atmospheres of N_2 or H_2 or H_2 plus I_2 and in some cases containing stoichiometric excess of Na or I_2 or containing various concentrations of Tl, have been explored from 2.5 to 8 ev. At -190°C an emission band centered at 295 mμ (4.20 ev) was observed in all but the one sample which contained the highest Tl concentration, 0.1 mole %. Another emission band at 375 mμ (3.31 ev) was found to be strongest in crystals containing excess I_2. A 425-mμ (2.92-ev) emission appeared strongly in all crystals containing Tl, but a 325-mμ (3.82-ev) band appeared only in the one crystal with 0.1 mole % Tl. With excitation energy below 5.5 ev the different emission bands arise from discrete excitation bands. With excitation energies above 5.5 ev up to 8 ev the emission was primarily in the 295-mμ band until Tl concentration reached 0.05 mole % where

the 425-mμ band and 295-mμ band became approximately equal. Maxima in reflection spectra coincide closely, but not exactly, with published maxima in the absorption spectra of NaI evaporated films. An inflection, not seen in the published absorption spectra, appears at 5.62 ev in the reflection spectrum at -190°C. Coincidental with the lowest energy maximum in the reflection spectrum at 5.59 ev, the excitation spectra for both 295-mμ and 425-mμ emission exhibit minima too strong to be accounted for by reflection loss. At excitation energies of 5.7 ev and above, luminescence quantum efficiency is greatest for an illuminating angle of incidence of 45°, whereas at lower energies no definitely preferable incidence angle was found. Also, with higher exciting energies it was found that the emission was strongest on the axis of illumination. With exciting energies below 5.5 ev the emission was isotropic.

MAGNETIC SUSCEPTIBILITY OF TETRAGONAL TITANIUM DIOXIDE. Frank E. Senftle, U. S. Geological Survey, Washington, D. C., Titus Pankey, Howard University, Washington, D. C. and Frank A. Grant, National Bureau of Standards, Washington, D. C. (Received July 1, 1960; revised manuscript received April 21, 1960).

Careful measurements have been made of the magnetic susceptibility of the rutile and anatase crystalline forms of titanium dioxide. The magnetic susceptibility of a single crystal of high-purity rutile was found to be $(0.067 \pm 0.0015) \times 10^{-6}$ emu per gram, and was temperature-independent from 55° to 372°K. Difficulty was encountered in obtaining a good value of the magnetic susceptibility of anatase because of impurities. However, a value of 0.02×10^{-6} emu per gram was obtained as a maximum value. A discussion is given for the different values obtained for anatase and rutile.

PHOSPHORESCENCE MECHANISMS. I. APPROACH AND GENERAL ANALYSIS. C. Billington, Division of Chemical Physics, Commonwealth Scientific and Industrial Research Organization, Chemical Research Laboratories, Melbourne, Australia (Received June 13, 1960).

A new technique is described for the study of decay mechanisms in phosphors. When a phosphor is excited by radiation whose intensity is modulated to a depth less than 10%, the modulation of the emission intensity has negligible harmonic content. Quantitative comparisons of these two modulated signals, if of sufficient accuracy and range, result in qualitative distinctions between different basic mechanisms, and the possibility of subsequent quantitative analysis. In particular, monomolecular and bimolecular processes are unequivocally distinguished.

PHOSPHORESCENCE MECHANISMS. II. DESCRIPTION OF PHOSPHOROMETER. C. Billington, Division of Chemical Physics, Commonwealth Scientific and Industrial Research Organization, Chemical Research Laboratories, Melbourne, Australia (Received June 13, 1960).

The functions of the phosphorometer are (i) to excite the phosphor under test with ultraviolet radiation whose intensity is modulated sinusoidally, (ii) to measure the modulation frequency, and (iii) to sample the excitation and emission intensities, and to extract from their modulated components the in-phase attenuation factor between them. The Hg arc source is current-modulated and stabilized by optical feedback over the range 3 to 30 000 cps. The average intensity of both excitation and emission is used to control the sensitivity of their respective photodetectors so that the modulation signals are always referred to the same datum. Both signals are passed in turn through a homodyne detector whose phase is adjusted to identity with the excitation modulation. The in-phase attenuation factor which is the ratio of the homodyne outputs is determined potentiometrically with a probable error of about 0.002.

PHOSPHORESCENCE MECHANISMS. III. METHOD OF ANALYSIS. C. Billington, Division of Chemical Physics, Commonwealth Scientific and Industrial Research Organization, Chemical Research Laboratories, Melbourne, Australia (Received June 13, 1960).

The method of deconvolution is discussed for the case in which phosphorescence is distributed between a number of concurrent components with different rate constants. The resolution of the method is deduced quantitatively from the known rms difference of the apparatus.

PHOSPHORESCENCE MECHANISMS. IV. DECAY RATE SPECTRA OF RUBY, URANIUM GLASS, AND Mn-ACTIVATED ZINC SULFIDE. C. Billington, Division of Chemical Physics, Commonwealth Scientific and Industrial Research Organization, Chemical Research Laboratories, Melbourne, Australia (Received June 13, 1960).

Ruby, uranium glass, and a ZnS:Mn sample were studied by means of the phosphorometer in a program of measurement and analysis which included variation of temperature and average exciting intensity. It was not possible to identify any laws of temperature dependence, but there is strong evidence that part of the phosphorescence mechanisms operating in the ZnS:Mn sample involves recombination.

THEORY OF IMPURITY RESISTANCE IN METALS. J. S. Langer, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received June 27, 1960).

A many-body technique is developed for the calculation of the dc resistivity of a Fermi fluid in the presence of a few, randomly scattered, fixed, impurities. A certain class of graphs yields an expression for the conductivity which is similar in form to the standard classical transport coefficient; but the decay time is determined by the scattering of single-particle-like excitations at the Fermi surface by screened impurities. A propagator method similar to that used in field theory is employed throughout the paper, and the perturbation-theoretic interpretation of this method is examined in some detail.

IMPURITY CONDUCTION AT LOW CONCENTRATIONS. Allen Miller* and Elihu Abrahams, Physics Department, Rutgers University, New Brunswick, New Jersey (Received June 23, 1960).

The conductivity of an n-type semiconductor has been calculated in the region of low temperature T and low impurity concentration n_D. The model is that of phonon-induced electron hopping from donor site to donor site where a fraction K of the sites is vacant due to compensation. To first order in the electric field, the solution to the steady state and current equations is shown to be equivalent to the solution of a linear resistance network. The network resistance is evaluated and the result shows that the T dependence of the resistivity is $\rho \propto \exp(\epsilon_3/kT)$. For small K, $\epsilon_3 = (e^2/\kappa_0)(4\pi n_D/3)^{1/3}(1 - 1.35K^{1/3})$, where κ_0 is the dielectric constant. At higher K, ϵ_3 and ρ attain a minimum near $K = 0.5$. The dependence on n_D is extracted; the agreement of the latter and of ϵ_3 with experiment is satisfactory. The magnitude of ρ is in fair agreement with experiment. The influence of excited donor states on ρ is discussed.

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FRICTIONAL STRESS ACTING ON A MOVING DISLOCATION IN AN OTHERWISE PERFECT CRYSTAL. Doris Kuhlmann-Wilsdorf, School of Metallurgical Engineering, University of Pennsylvania, Philadelphia, Pennsylvania (Received June 27, 1960).

The problem of the frictional stress suffered by moving dislocations in otherwise perfect crystals is investigated. This is done without calculating the core energies of dislocations, but by considering stresses and strains on the slip plane. The level of frictional stresses obtained is much higher than reported previously. Since common glide dislocations in metals with close-packed structures apparently do not suffer significant frictional stresses, mechanisms are discussed which tend to reduce their effect. A new such mechanism is discovered. It is based on the idea that the positions of dislocation axes are not defined with precision, but only within one to a few times the average displacement of the oscillating atoms. The expected result of this, is a depression of the frictional stress for close-packed metals even at very low temperatures, almost no effect on dislocations in crystals with diamond structures, and a temperature dependence proportional to $e^{-\text{const}\,T/T_M}$ for NaCl type salts and, probably, for bcc metals.

RELATION BETWEEN DIRAC AND CANONICAL DENSITY MATRICES, WITH APPLICATIONS TO IMPERFECTIONS IN METALS. N. H. March and A. M. Murray, Department of Physics, The University, Sheffield, England (Received June 17, 1960).

It is shown that the canonical density matrix in a single-particle framework may be related

directly to the generalized canonical density matrix, containing the Fermi-Dirac function, and to the Dirac density matrix.

A study is then made of density matrices in central field problems. A new differential equation is derived, from the Bloch equation, for the diagonal element of the canonical density matrix. In the case of a continuum of energy levels, this is shown to lead directly to a differential equation for the diagonal element of the Dirac matrix, that is the particle density. Free-electron density matrices are fully worked out and a perturbation theory based on these free-electron forms is presented.

It is further shown that for a nonspherical potential energy $V(\vec{r})$, the work of Green on the quantum-mechanical partition function may be utilized to yield a perturbation theory for the Dirac matrix. In this way, the correct formulation to replace Mott's well-known first order approximation for dealing with imperfections in metals is obtained. A brief discussion of the way in which this removes qualitatively the difficulties of the Mott treatment is given, and the possibility of direct numerical application in a self-consistent framework is pointed out.

DEPENDENCE OF SECONDARY ELECTRON EMISSION FROM MgO SINGLE CRYSTALS ON ANGLE OF INCIDENCE. A. B. Laponsky and N. R. Whetten, General Electric Research Laboratory, Schenectady, New York (Received November 24, 1959); revised manuscript received July 21, 1960).

The influence of the angle of incidence of primary electrons on the energy distribution and total yield of secondary electrons from MgO cleaved single crystals has been studied. Charging effects were minimized by using single-pulse techniques. The relative number of low-energy secondary electrons decreases with increasing angles of incidence. The effect appears to be most marked at primary energies in the vicinity of the maximum of the yield curve. At lower and higher primary energies the dependence on angle of incidence diminishes. Under most conditions the total yield increases with increasing angles of incidence. The magnitude of the increase is influenced by the primary electron energy. At low energies a slight decrease in total yield with increasing incidence angles has been observed. A dependence of the backscattered fraction of electrons on the angle of incidence of the primaries has been observed at several primary electron energies.

A number of maxima and minima occur superposed on the gross variation of total yield with angle of incidence. The angular positions of the maxima are insensitive to primary energy but show a dependence on the crystal orientation. Several models for accounting for the phenomena are discussed.

MECHANISM FOR COLORATION OF ALKALI HALIDES AT LOW TEMPERATURES. Clifford C. Klick, U. S. Naval Research Laboratory, Washington, D. C. (Received June 23, 1960).

A mechanism for low-temperature coloration of alkali halides is proposed in which double ionization of a halide ion or single ionization of two adjacent halide ions leads to the formation of an uncharged halogen molecule which occupies two adjacent halide ion sites. If the molecule is localized at one of these sites by the jump of a nearby halide ion into the other site, a vacancy and interstitial are formed. Capture of an electron by each of these imperfections leads to the production of the F center and H center.

NUCLEAR QUADRUPOLE RESONANCE IN SUPERCONDUCTING GALLIUM. R. H. Hammond and W. D. Knight, Department of Physics, University of California, Berkeley, California (Received June 13, 1960).

The nuclear quadrupole resonance (NQR) of Ga^{69} is investigated between 0.8 and 4.2°K. (The superconducting critical temperature is 1.084°K.) Progressive saturation of the resonance is produced at increasing rf power levels of a frequency-modulated marginal oscillator. The nuclear spin-lattice relaxation rate $1/T_1$ ($\sec^{-1}$) is approximately $\frac{1}{2}T$ (°K) in the normal state. The contact part of the hyperfine interaction appears to be predominant in producing relaxation. Although the saturation method does not permit a precise determination of the constant of proportionality, the comparison of normal and superconducting relaxation rates is considerably more reliable. The rf fields are assumed to obey the London law of penetration, and an average rf field is determined in the superconductor. The relaxation rate shows a maximum enhancement

by a factor 1.8 at approximately $0.95T_c$, a result which agrees with that obtained by Slichter and Hebel, and Redfield for the nuclear magnetic resonance of aluminum, and which serves as an additional experimental justification for certain features of the Bardeen-Cooper-Schrieffer theory of superconductivity. Unsaturated signal intensities in normal and superconducting states furnish a basis for estimating the penetration depth in superconducting metal spheres: $\lambda(0) = 1200$ A for an average particle diameter of 2.7 μ. The NQR frequency of Ga in the superconductor shifts by $(+)5\times10^{-5}$ of the normal frequency [the corresponding result for indium is found to be approximately $(-)10^{-2}$]. This means that the contribution of the conduction electrons to the average nuclear quadrupole coupling is modified by the rearrangement of the conduction band in the superconductor. It is demonstrated that, if the quadrupole term in the hyperfine interaction were to predominate, the spin relaxation rate in the superconductor would have a temperature dependence like that of the ultrasonic attenuation.

NEW METHOD FOR TREATING THE ANTIFERROMAGNETIC GROUND STATE. H. L. Davis,* Department of Physics and Astronomy, The Ohio State University, Columbus, Ohio (Received January 25, 1960; revised manuscript received June 22, 1960).

A perturbation expansion for the ground-state energy of an antiferromagnetic spin system is obtained in terms of a linked-spin-cluster expansion similar to Goldstone's linked-Feyman-diagram expansion for the interacting Fermion system. From the energy perturbation series, perturbation series for the long- and short-range order may be obtained. Using these perturbation series, the ground state properties are calculated through seventh order and compared with the results obtained by other investigators. In all cases, the values obtained here for the ground state energy are lower than those which have been obtained by purely variational means. The results for the long-range order are radically different from the variational results but agree qualitatively with those obtained by spin-wave theory; however, the method is free of the usual objections which are voiced to spin-wave treatments of antiferromagnetism. The present work is incomplete in that limits on the error introduced by using only a finite number of terms of the perturbation series to calculate the physical properties are not obtained. But the author feels that the merit of the present work is in the method rather than the results since it provides a consistent framework both to settle the convergence question and to treat the antiferromagnetic spin system at finite temperatures.

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VALLEY-ORBIT SPLITTING OF ANTIMONY IN GERMANIUM. H. Fritzsche, Department of Physics and Institute for the Study of Metals, University of Chicago, Chicago, Illinois (Received June 23, 1960).

The change of electrical conductivity under uniaxial tension and compression has been measured over the range 4°K to 7°K for single-crystal specimens of germanium doped with antimony. The stress was varied from 1×10^7 to 5×10^8 dynes/cm^2. On the basis of Price's calculation of the effect of shear on the Kohn-Luttinger donor level structure, an expression for the piezoresistance has been derived, which includes terms of high order in the strain. It is shown that for a finite valley-orbit splitting, i.e., a finite energy separation between the onefold and the threefold 1s-like donor states, shear increases the total electron concentration in the conduction band. For uniaxial stresses along the [110] direction this increase in electron concentration is an even function of stress and can, therefore, be determined from a linear combination of the piezoresistance measured under tension and compression. A comparison of the theoretical expression with the experimentally obtained change in electron concentration yields for antimony in germanium a valley-orbit splitting of 0.57 ± 0.03 milli-electron volt.

QUANTUM MECHANICAL LAW OF CORRESPONDING STATES FOR VAN DER WAALS SOLIDS AT 0°K. Newton Bernardes, Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa (Received June 22, 1960).

A quantum mechanical variational treatment based on a simple Heitler-London wave function is used to describe the various properties of

Van der Waals solids at 0°K. The effects of nuclear motion on the cohesive energy, volume, compressibility, sound velocity, etc., are discussed. These effects can be expressed in the form of a power series in $\hbar$, of which we show the first term, linear in $\hbar$, explicitly. The results are in good agreement with the available experimental data for all solidified inert gases, except for He to which the present method is not applicable.

ELECTRICAL RESISTIVITY OF THE HEAVY RARE-EARTH METALS. R. V. Colvin,* Sam Legvold, and F. H. Spedding, Institute for Atomic Research and Department of Physics, Iowa State University, Ames, Iowa (Received June 8, 1960; revised manuscript received August 1, 1960).

The electrical resistivities of polycrystalline Gd, Tb, Dy, Ho, Er, Tm, and Lu have been measured between 1.3°K and room temperature. The slope of the resistivity curve for Gd changes near the Curie point. The curve for Tb is very much like that for Gd but there is some evidence that two ordering temperatures exist for this metal. Dy, Ho, and Tm all show peaks in resistivity near their Néel points, while Er shows only a change in slope at its Néel point. The change from ferromagnetism to antiferromagnetism in Dy is seen as a sharp rise in the resistivity.

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EFFECT OF POINT IMPERFECTIONS ON LATTICE THERMAL CONDUCTIVITY. Joseph Callaway, Division of Physical Sciences, University of California, Riverside, California, and Hans C. von Baeyer, Department of Physics, University of Miami, Coral Gables, Florida (Received June 1, 1960).

The consequences of a simple, phenomenological, theory of lattice thermal conductivity with respect to the effect of point imperfections are summarized. The experimental results of Berman et al. on the effect of varying the concentration of Li^6 on the conductivity of lithium fluoride are analyzed in detail.

THEORY OF THE COMPLEX REFRACTIVE INDEX. C. Alden Mead, School of Chemistry, University of Minnesota, Minneapolis, Minnesota (Received February 12, 1960).

A quantum-mechanical formalism is developed for representing the complex refractive index of a gas, including both dispersive and absorptive parts. In analogy with some previous work by the author, it is shown that the phenomenological Maxwell equations are obeyed as equations between certain matrix elements of operators representing the field quantities. The method makes use of a canonical transformation due to Arnous and Bleuler, and also of the "damping operators" previously introduced by the author. It is shown that the calculation of the complex refractive index reduces to the calculation of a single diagonal matrix element of a damping operator. The actual evaluation of this matrix element is taken up in the following paper.

RESONANCE BROADENING OF SPECTRAL LINES. C. Alden Mead, School of Chemistry, University of Minnesota, Minneapolis, Minnesota (Received February 12, 1960).

A formalism introduced previously by the author is used to calculate the shape of an absorption line in a gas, taking account of the "resonance" interaction between like atoms. The most serious approximation is the neglect of the effect of translational motion. An asymmetric line is obtained, with the maximum shifted slightly toward the red. Some simple classical considerations also agree qualitatively with the results. Various properties of the line shape obtained are briefly discussed. The neglect of translational motion makes it impossible to compare the results with experiments performed up to the present time.

ELASTIC AND INELASTIC SCATTERING OF ELECTRONS FROM THE S STATES OF ATOMIC HYDROGEN. Kenneth Smith, Argonne National Laboratory, Argonne, Illinois (Received May 19, 1960; revised manuscript received August 1, 1960).

Total elastic and inelastic cross sections, for the scattering of electrons (including exchange) from the 1s, 2s, and 3s states of atomic hydro-

gen, have been calculated in three approximations using the method of partial waves. The purpose of this report is to present the S-, P-, and D-wave contributions to the elastic, $Q_{1s\text{-}1s}$, and inelastic, $Q_{1s\text{-}2s}$, cross sections.

METASTABLE HYDROGEN MOLECULES. William Lichten, Ryerson Laboratory, Department of Physics, University of Chicago, Chicago, Illinois (Received June 30, 1960).

The existence of a metastable electronic state of H_2 has been established by means of the molecular beam magnetic resonance technique. Observations have been made of the radio-frequency Zeeman spectrum of natural and para-hydrogen. At a magnetic field of about 5 gauss the strongest line corresponds to a g value of 0.83. The appearance potential for this line is 11.9 ± 0.25 ev. Observation of the quadratic Zeeman effect made it possible to estimate the fine structure of the $N=2$ level of para-hydrogen. The observed intervals are: $(J=2 \rightarrow J=3) = 5730 \pm 200$ Mc/sec, $(J=2 \rightarrow J=1) = 5000 \pm 200$ Mc/sec. This represents the first complete resolution of the fine structure of a rotational level of H_2. Knowledge of the electronic structure of H_2, agreement of the appearance potential with spectroscopic data, agreement of the fine structure with partially resolved optical spectra, and good agreement of the Zeeman spectra with theory lead to the conclusion that the metastable state must be H_2, $c(1\sigma_g, 1\pi_u)\ ^3\Pi_u$.

HYPERFINE STRUCTURE OF HYDROGEN, DEUTERIUM, AND TRITIUM. L. Wilmer Anderson* and Francis M. Pipkin, Lyman Laboratory, Harvard University, Cambridge, Massachusetts, and James C. Baird, Jr.,† Mallinckrodt Laboratory, Harvard University, Cambridge, Massachusetts (Received June 3, 1960).

The optical transmission of an optically oriented rubidium vapor in spin-exchange equilibrium with atomic hydrogen, deuterium, and tritium has been used to measure with high precision the hyperfine splittings of these paramagnetic atoms. The results are $\Delta\nu(H) = 1420.405726 \pm 0.000030$ Mc/sec, $\Delta\nu(D) = 327.384349 \pm 0.000005$ Mc/sec, and $\Delta\nu(T) = 1516.701396 \pm 0.000030$ Mc/sec. These results are based on a value of the hyperfine splitting of Cs^{133} which is taken to be $\Delta\nu(Cs^{133}) = 9192.631840$ Mc/sec.

These measurements were made in various buffer gases which caused a shift in the observed hyperfine splitting, and the results given represent extrapolations to zero pressure. The pressure shifts were measured for H in argon, neon, helium, and molecular hydrogen and were measured for D and T in argon and neon. The assigned limits of error represent the range of disagreement of the zero-pressure extrapolations in the different buffer gases.

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APPLICABILITY OF ROOTHAAN'S SELF-CONSISTENT FIELD THEORY. S. Huzinaga,* Laboratory of Molecular Structure and Spectra, Department of Physics, University of Chicago, Chicago, Illinois (Received June 28, 1960).

Roothaan's open-shell self-consistent field theory is extended in such a way that it can cover a number of important classes of atomic and molecular states which are not included in the range of applicability originally claimed.

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POSITRONIUM DECAY IN MOLECULAR SUBSTANCES. Werner Brandt, Radiation Physics Laboratory, Engineering Department, E. I. du Pont de Nemours and Company, Incorporated, Wilmington, Delaware, and S. Berko and W. W. Walker, Department of Physics, University of Virginia, Charlottesville, Virginia (Received March 16, 1960).

The overlap of the positron component of the Swiss-cheese wave function of positronium (Ps) in dense molecular substances with the lattice wave function determines the 2γ annihilation rate of positrons bound in orthopositronium via electron pickup from the lattice (τ_2 decay), and in part the 2γ angular correlation. Ps acts as a probe for the dependence of one-particle wave functions in lattices on structural lattice parameters, and of radical-molecule interactions on atomic lattice parameters. To explore the sensitivity of this probe Ps wave functions have been calculated for different lattice structures in the Wigner-Seitz approximation, and the dependence of τ_2 decay on lattice parameters derived. This

dependence was tested experimentally by τ_2 measurements on carefully characterized molecular substances in different physical states. It is confirmed that the temperature and state dependence is primarily a "free volume" effect in the sense that the overlap integral between the Ps and lattice wave functions decreases with increasing lattice spacing. The effects of a spherical confinement on the self-annihilation rates and stability of Ps are shown to be small for radii relevant to molecular lattices.

An anomalous behavior is found for the ice-water transition, where τ_2 increases significantly despite the contraction of water between the melting point and 4°C.

K-ELECTRON CAPTURE BRANCH OF Sr^{87m}. A. W. Sunyar and M. Goldhaber, Brookhaven National Laboratory, Upton, New York (Received June 29, 1960).

The existence of a $(0.65 \pm 0.25)\%$ *K*-electron capture branch in the decay of 2.8-hr Sr^{87m} has been established by means of an "internal source" technique. The low $\log ft = 4.25$ and small energy release of 115 kev make this *K* branch especially interesting because it establishes Rb^{87} as a possible low-threshold detector for the inverse neutrino capture reaction, $Rb^{87} + \nu \rightarrow Sr^{87m} + e^-$, which could be identified by observing the characteristic decay of the Sr^{87m} isomer.

RADIOACTIVE DECAY OF Lu^{170}. R. G. Wilson and M. L. Pool, Department of Physics and Astronomy, Ohio State University, Columbus, Ohio (Received April 4, 1960).

Ytterbium oxide enriched to 81.4% in the 170 mass number was irradiated with 6-Mev protons. An activity decaying by electron capture with a half-life of (2.05 ± 0.05) days was produced and assigned to Lu^{170} by the identification of the ytterbium *K* x ray and by comparison with the activities produced by similar proton irradiations of each of the other enriched isotopes of ytterbium. The observed activity consists of the ytterbium *K* x ray, gamma rays with energies of 84, 193, 245, 1010, 1030, 1165, 1275, 1415, 2035, 2365, 2665, 2890, and 3085 kev, and a small amount of annihilation radiation. Because no particle radiation exists, the mode of decay of Lu^{170} is solely by electron capture to Yb^{170}. Gamma-gamma coincidence measurements have led to the postulation of levels in Yb^{170} at 2120 (0), 2365 (1 or 2), 3170 (3), and 3395 (1 or 2) kev in addition to the previously assigned 84 (2+), and 278 (4+) levels. A partial energy level scheme with approximate electron-capture branching ratios is proposed.

APPROXIMATE CALCULATION OF NUCLEAR BINDING ENERGY. Leonard S. Rodberg and Vigdor L. Teplitz, University of Maryland, College Park, Maryland (Received June 16, 1960).

The binding energy, size, and shape of finite nuclei are studied using a method which tests the "local uniformity" assumption. This assumption implies that the properties of finite nuclei can be obtained from those of infinite nuclear matter. The energy is computed using as a trial wave function an amplitude- and frequency-modulated plane wave; this permits the use of nuclear-matter results in a straightforward manner. The calculation gives a binding energy which is too low, a radius which is too small, and a nuclear surface which is too diffuse. Several possible corrections to this result are examined.

ATOMIC BEAM STUDY OF THE HYPERFINE STRUCTURE OF THULIUM-170. Amado Y. Cabezas and Ingvar Lindgren,* Lawrence Radiation Laboratory, University of California, Berkeley, California (Received June 3, 1960).

The atomic-beam magnetic-resonance technique has been used to measure atomic and nuclear quantities of the 129-day isotope Tm^{170} as follows: $J = 7/2$, $g_J = 1.14122 \pm 0.00015$, $I = 1$, $|A| = 200 \pm 3$ Mc/sec, and $|B| = 1010 \pm 15$ Mc/sec. The values of J and g_J are consistent with the ground-state assignment $^2F_{7/2}$. Values of the nuclear moments are calculated from the hyperfine-structure interaction constants A and B by use of a two-parameter radial wave function, in which one parameter is determined from comparison with Hartree functions and the other parameter from the experimental spin-orbit coupling constant. Uncorrected values are obtained, as follows: $|\mu_I| = 0.26 \pm 0.02$ nm and $|Q| = 0.61 \pm 0.05$ barn, with the two moments of the same sign. The same wave function is used to calculate the relativistic and diamagnetic corrections to the atomic g value,

and the result is in excellent agreement with the experiment.

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PREFORMATION FACTOR IN EMISSION OF COMPLEX PARTICLES FROM NUCLEAR REACTIONS. Bernard L. Cohen, University of Pittsburgh, Pittsburgh, Pennsylvania (Received May 27, 1960; revised manuscript received July 11, 1960).

The resason why complex particles (alphas, tritions, Li^6 nuclei, etc.) are emitted in nuclear reactions as frequently as nucleons (after corrections for Coulomb barrier penetration and energetics), whereas ice crystals are never emitted from evaporating water droplets, is investigated. It is shown that the difference is entirely explained by the fact that a nucleus is a highly degenerate system subject to Fermi-Dirac statistics, whereas an evaporating water droplet is a nondegenerate statistical system.

NEW NUCLIDE, KRYPTON-74. J. H. Gray, III, H. L. Hamester,* and A. A. Caretto, Jr., Department of Chemistry, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received June 13, 1960).

A new nuclide of krypton, Kr^{74}, has been produced by high-energy proton interaction with strontium. It was found to have a half-life of (20 ± 1) minutes and to decay by (3.10 ± 0.1)-Mev positrons. No gamma rays in the energy range 0.4 to 4.0 Mev were observed. Evidence for the production of Kr^{75} was also observed. Calculations indicate an upper limit for the half-life of (0.8 ± 0.4) minute.

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ASYMMETRY IN THE ELASTIC SCATTERING OF POLARIZED PROTONS BY DEUTERIUM. Stewart M. Marcowitz, The Enrico Fermi Institute for Nuclear Studies and Physics Department, The University of Chicago, Chicago, Illinois (Received June 3, 1960).

Asymmetries in the elastic scattering of polarized protons by deuterium have been measured at 419 Mev at proton center-of-mass angles from 47.9° to 115°. The inferred polarizations are: -0.092 ± 0.058 at 47.9°, -0.502 ± 0.058 at 59°, -0.583 ± 0.081 at 68°, -0.128 ± 0.046 at 77.8°, -0.122 ± 0.111 at 90°, -0.141 ± 0.056 at 97°, -0.309 ± 0.073 at 105°, and $+0.174 \pm 0.059$ at 115°. It is suggested that using the impulse approximation these results may serve as a test of nucleon-nucleon scattering phase shifts or, with proton-proton scattering data, may be used to obtain $I = 0$ nucleon-nucleon phase shifts.

RARE $E2$ TRANSITION IN C^{13}. R. E. Pixley, J. V. Kane, and D. H. Wilkinson,* Brookhaven National Laboratory, Upton, New York (Received June 22, 1960).

By means of a coincidence method it is shown that the 5/2+ state of C^{13} at 3.86 Mev has an $E2$ branch of relative strength $(9.3 \pm 2.0) \times 10^{-3}$ to the $\frac{1}{2}$+ state at 3.09 Mev. From consideration of the likely strengths of the competing $E1$ transition to the 3/2- state at 3.68 Mev and the $M2$ transition to the ground state, it is deduced that this branching ratio implies an $E2$ transition speed of the order of a single-particle (proton) unit. This in turn demands the substantial participation of at least one excited state of C^{12} in the parentage of at least one of the C^{13} states with respect to the $1d$ or $2s$ neutron. This eliminates in particular a jj-coupling description of C^{13} in which the two states in question are $1d_{5/2}$ and $2s_{1/2}$ neutron states and more generally a weak-coupling model in which the ground state of C^{12} is the unique parent for the C^{13} states (with respect to $1d$ and $2s$ neutrons).

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MEASUREMENT OF THE $O^{16}(n,p)N^{16}$ CROSS SECTION AT 14.7 Mev. J. A. DeJuren and R. W. Stooksberry, Bettis Atomic Power Laboratory, Westinghouse Electric Corporation, Pittsburgh, Pennsylvania (Received June 22, 1960).

The $O^{16}(n,p)N^{16}$ cross section has been measured for 14.7-Mev neutrons. A recoil proton telescope was used to measure the neutron flux. Oxygen was incorporated in a liquid scintillator which was irradiated in the flux. The absolute activity of the resulting N^{16} was measured by 4π beta counting the solution with two photomulti-

pliers in coincidence. A value of 39.2 ± 1.6 mb was obtained for the cross section.

EXTENSION OF THE SHELL MODEL FOR HEAVY SPHERICAL NUCLEI. Michel Baranger, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received June 17, 1960).

The Bardeen-Bogoliubov-Belyaev treatment of the pairing correlations is applied to spherical nuclei with a general nuclear force. The interaction between quasi-particles is treated by the method of linearized equations of motion. An advantage of this treatment is that the same equations describe single-particle excitations and collective excitations, so that the former are orthogonal to the latter and the total number of states is correct. Another advantage is that the spurious states due to the fluctuations in the number of particles are automatically eliminated. The equations to be solved resemble those for a two-body shell model calculation. Simple estimates, based on delta-function or quadrupole forces, are made for the vibrational frequencies in various modes and transition matrix elements. It is concluded that the method is as powerful as other known methods for dealing with collective states by the shell model, and that the same order of magnitude for the effective nuclear force seems capable of fitting all the data.

COMPARISON OF Po^{210} ALPHA-PARTICLE ENERGY WITH THE $Li^7(p,n)Be^7$ REACTION THRESHOLD ENERGY. C. P. Browne, J. A. Galey, J. R. Erskine, and K. L. Warsh, Department of Physics, University of Notre Dame, Notre Dame, Indiana (Received June 17, 1960).

Recent absolute measurements of the Po^{210} alpha-particle energy disagree with the older value used as the standard for many nuclear reaction energy measurements. A new comparison with the $Li^7(p,n)Be^7$ reaction threshold energy was made using the Notre Dame electrostatic accelerator and broad-range spectrograph. Four separate methods of comparison were used. In the first three the threshold was run and then protons or deuterons were scattered from appropriate targets so that the scattered group was recorded on the spectrograph plate near the alpha group from a source placed at the target position. First, the spectrograph and, second, the beam analyzer were used to compare particle momenta. Third, with both fields held constant after the threshold was run with the molecular beam, deuterons were scattered, giving particles of the same $B\rho$ as the alphas. In the fourth method several reaction energies that are precisely known in terms of the $Li^7(p,n)Be^7$ reaction threshold energy were measured in terms of the Po^{210} alpha-particle energy. These were the $Mg^{24}(d,d')Mg^{24*}$ reaction to the first excited state of Mg^{24} and the $N^{14}(d,p)N^{15}$ reaction leading to three excited states of N^{15}. The four measurements agree and give 5.3086 ± 0.003 Mev for Po^{210} alpha-particle energy based on 1.8811 Mev for the $Li^7(p,n)Be^7$ reaction threshold energy.

jj-COUPLING MODEL IN ODD-ODD NUCLEI. M. H. Brennan* AND A. M. Bernstein, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received June 10, 1960).

An analysis has been made, using the *jj*-coupling model, of the spins of 76 low-lying levels of odd-odd nuclei with $20 < A < 120$. Levels have been excluded from the analysis if any ambiguity exists in the assignment of a configuration. Excluding particle-hole configurations, we find Nordheim's "strong" rule obeyed in the 22 cases in which it is applicable. Nordheim's "weak" rule is replaced by a rule predicting a ground state of the highest or lowest allowable spin. This revised rule is obeyed in 38 out of 41 cases. This competition between two levels of greatly differing spin results in the frequent occurrence of isomerism. For particle-hole configurations the state of the highest spin minus one is the ground state in 6 out of 13 cases. This agreement with experiment is obtained using proton and neutron configurations which, in 66 of the 76 cases, are found as ground states in the neighboring odd-even nuclei. The three revised coupling rules are predicted by the calculations of Schwartz, in which the residual proton-neutron interaction has a delta-function radial dependence, if the singlet-to-triplet strength is taken as 0.6, independent of mass number. This value is the same as that required to fit the free two-nucleon data. Exceptions to the empirical coupling rules and this theory will be discussed.

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RECOIL PROPERTIES OF FISSION PRODUCTS. John M. Alexander and M. Frances Gazdik, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received May 16, 1960).

The range and relative rates of energy loss in Al and Au have been measured radiochemically for five products from thermal-neutron-induced fission of U^{235}. Range-velocity relationships for the median light product of fission and the median heavy product have been obtained from these measurements and other workers' energy-loss data. The relation of range (R) to energy (E) or velocity (V) can be fitted to functions of the form $R=kV-\Delta$ or $R=KE^{2/3}$. We have assumed that these functional forms can be applied to fission products of any mass. The constants K and Δ were determined from values of the range and kinetic energy for products of high yield. The values of these constants have been extrapolated to products of low yield. We have estimated kinetic energies, heretofore unmeasured, from the ranges of low-yield products.

We have interpreted certain radiochemical observations in terms of the average component of the range perpendicular to the original velocity. The value of this component in Au has been estimated to be about one-fifth the total range.

ISOMERS OF Am^{242}. F. Asaro, I. Perlman, J. O. Rasmussen, and S. G. Thompson, Lawrence Radiation Laboratory and Department of Chemistry, University of California, Berkeley, California (Received June 9, 1960).

New measurements on the Am^{242} isomers show that the 16-hr activity is the ground state and the 152-yr activity the upper isomer, contrary to earlier evidence. By electrostatic collection techniques some 16-hr americium activity was separated from an aged sample containing the 152-yr isomer. Careful study of the conversion electron spectrum of the sample containing 152-yr Am^{242m} revealed eleven conversion lines of a 48.6-kev transition converted in americium, and the relative M-subshell conversion intensities are only consistent with an $E4$ assignment. This information, coupled with the recent atomic beam measurement of spin 1 for the 16-hr activity, leads to a spin assignment of 5 for Am^{242m}.

The experimental information on these isomers is given a detailed analysis in terms of the Bohr-Mottelson model. The most surprising conclusion of this analysis is that the spin 1 ground state has a K-quantum number of zero, and hence a negative spectroscopic quadrupole moment. Data on levels and decay of the low-spin isomers of Ho^{166} and Ta^{180} are also considered and lead to the conclusion that there is generally an energy displacement between odd and even spin members of $K=0$ rotational bands in odd-odd spheroidal nuclei.

NUCLEAR ORIENTATION OF Mn^{54} AND Mn^{52m} Rudolf W. Bauer, Martin Deutsch, Gordon S. Mutchler, and Donald G. Simons,* Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received June 17, 1960).

The spins of the nuclides Mn^{54} (290 days) and Mn^{52m}(21 min), incorporated in the lattice of cerium magnesium nitrate and nickel fluosilicate crystals, have been oriented at low temperatures. The measurement of the angular distribution of the gamma rays from Mn^{54} indicate the spin of Mn^{54} to be 3 or 2; in the latter case, the beta decay must be predominantly of the Fermi type with an upper limit of 10% of a possible Gamow-Teller admixture. From a simultaneous measurement of the angular distribution of the gamma rays from Mn^{54} and 5.7-day Mn^{52} the ratio of the nuclear g values of the two isotopes is found, which yields a magnetic moment for Mn^{54} of 2.55 ± 0.21 nm (for spin 3) or 2.16 ± 0.26 nm (for spin 2). The circular polarization measurements of the gamma rays from Mn^{54} determine this moment to be positive. Similar angular distribution experiments on Mn^{52m}, assuming spin 2, yield a magnetic moment of 1.04 ± 0.16 nm if the beta decay is predominantly a Gamow-Teller transition, or 0.72 ± 0.16 nm for a pure Fermi transition. These results indicate that the nuclear g values of the 21-min and the 5.7-day states of Mn^{52} are about the same.

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ASYMMETRY IN 143-Mev pn SCATTERING. Arthur F. Kuckes* and Richard Wilson, Cyclotron Laboratory, Harvard University, Cambridge, Massachusetts (Received June 16, 1960).

A beam of 145-Mev protons of polarization 69 ± 2% was scattered from liquid deuterium.

pn scattering events were identified by a coincidence method. The asymmetry in the scattering was measured. These data can be compared with polarization in free np scattering at the appropriate angles.

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ELASTIC SCATTERING OF 5-Bev π^- MESONS ON HYDROGEN. Richard G. Thomas, Jr., Lawrence Radiation Laboratory, University of California, Berkeley, California (Received June 9, 1960).

The elastic scattering of 5-Bev negative pions on hydrogen has been observed in a large propane bubble chamber. We report below the results of this observation. It is shown that $\simeq(7\pm3)\%$ of the events called elastic are probably background. The observed angular distribution is strongly peaked forward. This fact suggests that diffraction scattering is the dominant process at this energy. The theoretical analysis of the distribution is in terms of the optical model. It is shown that the proton acts like a partially opaque sphere of radius 1.04×10^{-13} cm $\pm5\%$. The total elastic cross section is found to be 5.6 ± 0.5 mb. From the extrapolated value of $d\sigma(0)/d\Omega=29.8$ mb/sr $\pm10\%$ in the center-of-mass system, a value of 29.1 ± 2.9 mb was calculated for the total hydrogen cross section. The opacity of the sphere is thus 0.69 ± 0.05.

INTERACTION OF K^+ MESONS WITH EMULSION NUCLEI; K^+-ENERGY RANGE 140 - 375 Mev. B. Sechi-Zorn and G. T. Zorn, Brookhaven National Laboratory, Upton, New York (Received June 6, 1960).

Experimental data are presented on the interaction in nuclear emulsions of K^+ mesons in the energy range 140 - 375 Mev.

From an optical-model calculation based on a Wood-Saxon potential ($r_0=1.15A^{1/3}$ f, $d=0.57$ f), the real K-nuclear potential V was found to be positive (or repulsive), varying from 18.5 ± 3.3 Mev at the lowest energy, to 13.5 ± 5.0 Mev at the highest energy. The value of the imaginary nuclear potential W changed with increasing energy from -12.9 ± 1.4 Mev to -17.6 ± 2.5 Mev.

The K-nucleon cross section was derived from the imaginary nuclear potential and, under the assumption of the independent-particle model of the nucleus, the total K-neutron non-charge-exchange cross section σ_{Kn^+} and the charge-exchange cross section σ_{Kn^0} were determined. It was found that σ_{Kn^+} decreases with increasing energy from 12.1 ± 1.6 to 8.1 ± 1.5 mb, whereas σ_{Kn^0} increases from 4.6 ± 1.2 mb to 10.3 ± 2.4 mb.

A phase-shift analysis using the total cross sections σ_{Kp^+} (the K-proton cross section), σ_{Kn^0}, $\bar{\sigma}_{ub}$ (the average K-nucleon cross section), and V gave evidence for P-wave contribution in isotopic spin state $T=0$. Changes in the K-neutron integrated cross sections with increasing K^+ energy seem to be accompanied by a change from a predominantly forward or near symmetric to a predominantly backward center-of-mass angular distribution for the charge-exchange K-neutron scattering. The opposite follows for the non-charge-exchange K-neutron differential cross section.

Charged π-meson production by K^+ mesons has been observed, and the K-nucleon cross section at ~280 Mev K^+ energy was estimated to be ~1/7 mb.

EXPERIMENTAL STUDY OF THE MAGNETIC STRUCTURE OF THE NEUTRON. J. I. Friedman, H. W. Kendall, and P. A. M. Gram, High-Energy Physics Laboratory, Stanford University, Stanford, California (Received June 16, 1960).

A measurement of the ratio of the magnetic form factor of the neutron to that of the proton has been carried out by comparing large- and small-angle elastic electron-deuteron scattering at constant four-momentum transfers. The experimental result for the average value of the ratio in the range of momentum transfers from 1.6 f^{-1} to 2.25 f^{-1} is $F_{1n}/F_p=(0.91\pm0.05)\pm0.07$; the first error is a standard deviation from experimental uncertainties, and the second from theoretical uncertainties in the analysis. Measurements of the ratio of the nucleon isotopic scalar form factors have also been obtained from this experiment. The average value of F_2^S/F_1^S for the same range of momentum transfers has been found to be $(+0.06\pm0.09)\pm0.13$. The small-angle scattering data have been used to determine the charge form factor of the deuteron in the range of momentum transfers from 0.98 f^{-1} to 2.8 f^{-1}. The results are consistent with a repulsive-core model of the deuteron.

DETERMINATION OF THE FORWARD SCATTERING AMPLITUDE FROM THE OPTICAL MODEL POTENTIAL IN THE HIGH-ENERGY LIMIT. R. Lipperheide and D. S. Saxon, Department of Physics, University of California, Los Angeles, California (Received June 6, 1960).

A relation between the two-particle forward scattering amplitude, $f_k(0)$, and the volume integral over the optical potential, $V_k(\vec{x})$, is given in the high-energy limit, which is valid at least to second order in the two-body interactions, neglecting correlation and exchange terms. The relation is

$$\int V_k(\vec{x})d\vec{x} = -\frac{2\pi\hbar^2}{m}Af_k(0),$$

where A is the number of nucleons in the target, and m is the reduced mass of the two-particle system. The connection between the shape of the optical model potential and the density distribution of the target nucleus is discussed.

SEARCH FOR $S=-2$ NEGATIVE HEAVY MESONS IN NUCLEAR EMULSIONS. Y. Eisenberg and M. Friedmann, The Weizmann Institute of Science, Rehovoth, Israel, and G. Alexander and D. Kessler, Israel Atomic Energy Commission Establishment, Rehovoth, Israel (Received June 24, 1960).

A search for the D^- meson was conducted in an emulsion stack exposed to the 300-Mev/c K^- beam at Berkeley. The stars initiated by the beam particles were investigated for possible emission of two strangeness-carrying particles, and in addition mass measurements were done on the short-range tracks. In about 10 000 events examined, no possible $S=-2$ particle has been observed. Hence the upper limit for the D^--meson occurrence in the present experimental conditions is ~1/100 from the double strangeness observation and ~1/6000 from direct mass determinations.

MULTIPLE MESON PRODUCTION IN NUCLEON-ANTINUCLEON ANNIHILATIONS. LeRoy F. Cook, Jr.* and Joseph V. Lepore, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received April 6, 1960).

A two-parameter model is proposed for treating complicated production problems in a relatively simple way. It is assumed that the interaction may be characterized by a range of interaction and by a coupling strength. After the model is developed, it is applied to the problem of pion production in N-$\bar{N}$ annihilations. The two parameters are fixed by the experimental data for the multiplicity and energy spectra. It is found that all the data can be satisfied if one chooses the radius of interaction to be one pion Compton wavelength. Under certain restrictions the model reduces to the Fermi model.

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INTERNAL PAIRS FOLLOWING π^- CAPTURE IN HYDROGEN. M. Derrick, J. G. Fetkovich, T. H. Fields, and J. Deahl, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received June 17, 1960).

Internal electron pairs from the two reactions (1) $\pi^-+p\to n+\pi^0\to n+\gamma+e^++e^-$ and (2) $\pi^-+p\to n+e+e^-$ have been studied in a hydrogen bubble chamber. 2184 cases were seen. A geometrical cutoff selected 1523 of these as suitable for momentum measurement. By an analysis of the momentum spectrum the Panofsky ratio was measured to be 1.51 ± 0.10. The total intensity, momentum partition within the pairs, and distribution in virtual photon mass are in essential agreement with the theoretical predictions of Kroll and Wada as recently extended by Joseph.

RELATIVE IONIZATION OF PROTONS NEAR THE END OF THEIR RANGE. Hans Bichsel, Department of Physics, University of Southern California, Los Angeles, California (Received June 13, 1960).

The influence of multiple scattering on straggling and consequently on the ionization produced by protons close to the end of their range is derived. The effect due to nuclear reactions is shown to be small at 340 Mev. The observed data at this energy are satisfactorily accounted for.

PION-NUCLEON SCATTERING IN THE MANDELSTAM REPRESENTATION. S. C. Frautschi, Department of Physics, University of California, Berkeley, California, and J. D. Walecka, Insti-

tute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California (Received June 6, 1960).

Using the analytic properties of partial wave scattering amplitudes, as derived from Mandelstam's representation, we have studied the $J=3/2$, P state of the pion-nucleon system. The method used is covariant, it incorporates unitarity, and the effect of a possible pion-pion resonance has been investigated. Using the "single nucleon term" and the low-energy scattering properties of the "crossed states," we obtain a resonance in the $J=3/2$, $T=3/2$ pion-nucleon state without the aid of a cutoff. We have also investigated the scattering in the $T=1/2$ state. The pion-pion resonance appears to have only a very small effect in the $T=3/2$ state, whereas in the $T=1/2$ state it increases the phase shift by a factor of 2.

The resonance obtained in the $T=3/2$ state occurs at too low an energy. There are several factors which may account for this: We have not been able to include fully the contributions from crossed states, and we have not systematically included inelastic scattering.

GENERALIZED ANGULAR MOMENTUM IN MANY-BODY COLLISIONS.

Felix T. Smith, Stanford Research Institute, Menlo Park, California (Received May 2, 1960).

With short-range forces, initial and final states in a classical 3-body collision are straight-line trajectories into and out of a region where all three particles are close together at the same time. Using six coordinates, three describing the relative position of a pair of particles, and three the relative position of the third particle and the center of mass of the pair, the condition for simultaneous togetherness can be expressed with the help of the 6×6 grand angular momentum tensor, Λ, whose components are $\Lambda_{ij}=(m_i/m_j)^{1/2}\times x_i p_j-(m_j/m_i)^{1/2}x_j p_i$. For a close 3-body collision, $\Lambda^2=\frac{1}{2}\sum_{i,j}\Lambda_{ij}^2$ must be small. Λ^2 commutes with the ordinary angular momentum operators and with the kinetic energy; its eigenvalues are $\lambda(\lambda+4)\hbar^2$, with integral λ, and its eigenfunctions hyperspherical harmonics. Initial and final 3-body states can be described quantally by the total energy E, Λ^2, and a commuting set of ordinary angular momenta; this description has the same relation to a momentum representation as the ordinary angular momentum analysis has for a 2-body collision. A collision of $(N+1)$ particles can be described by using a hierarchy of operators Λ_n^2 $(2\leq n\leq N)$; their eigenvalues are $\lambda_n(\lambda_n+3n-2)\times\hbar^2$.

REMARKS ON SCHRÖDINGER'S MODEL OF DE SITTER SPACE.

W. Rindler, Department of Mathematics, Cornell University, Ithaca, New York (Received June 6, 1960).

Schrödinger in his book "Expanding Universes" developed a "reduced model" of de Sitter space-time consisting of a certain hyperboloid of one sheet. He showed, inter alia, that timelike sections of the hyperboloid by planes through its center correspond to free paths in de Sitter space-time. The main purpose of the present note is to show that timelike sections by arbitrary planes correspond to paths of uniformly accelerated particles, and then to deduce some simple properties of such paths.

ABSENCE OF BOUND STATES IN A GRAVITATIONAL FIELD.

Asher Peres, Department of Physics, Israel Institute of Technology, Haifa, Israel (Received June 30, 1960).

The Klein-Gordon and Dirac equations for a particle in the gravitational field of a point mass are investigated. It is shown that the geometrical properties of the Schwarzschild metric prevent the normalization of any bound state solutions.

QUENCHING OF MAGNETIC MOMENTS IN NUCLEI.

S. D. Drell and J. D. Walecka, Department of Physics and Institute of Theoretical Physics, Stanford University, Stanford, California (Received June 20, 1960).

Starting from the premise that with modern dispersion-theoretical techniques one has a reliable method for calculating the anomalous magnetic moment of a nucleon, we have calculated the modification or "quenching" of this moment for a nucleon in nuclear matter. The effect we consider here is due to the fact that nucleons are not allowed by the exclusion principle to recoil into states already occupied by other nucleons in the nucleus. The actual technique we have used in our calculation is to sum all the Feynman diagrams that are included in the dispersion theory calculation of the single-